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Mixed Numerical-Experimental Identification Based on Modal Testing

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Abstract—The discrete mass distribution of metallic foam causes discontinuities in their local properties. Mechanical properties of metallic foams depend on their relative densities. A mixed numerical-experimental identification technique is used to identify elastic behavior of Aluminum Foam. The developed identification technique is based on experimental modal model, measured computer tomography density distribution data and FE model. Experimental and numerical eigen values and their corresponding mode shapes are used for identification of elastic behavior. A set of error functions based on measured and numerical eigen frequencies and corresponding mode shapes was developed in MATLAB. The aim of this work is to develop and validate a mixed Numerical-experimental technique for identification of elastic behavior of Aluminum foam.

Index Terms—MNET, ANSYS, Identification technique, Inverse Problem, Modal testing.

I. INTRODUCTION

The MNETs have proven to be very versatile and flexible tools for identification of material parameters. MNET-based identification routines have been introduced in a wide range of disciplines to estimate a broad variety of material parameters. Various MNETs have been developed till now. They are distinguished primarily by the techniques used for optimization of parameters, the definition of the functional error, the kind of experiments, by the type of experimental data used or the modelling and its numerical solution implementation. The accuracy and convergence properties of these methods depend mainly on the correlation between the experimental conditions and their numerical simulations (consistency of the model and experiment), the sensitivity of numerical solutions of the model parameters to identify, the sensitivity of the error functional measuring the numerical-experimental correlation and the performance of the adopted optimization technique. Great effort has been devoted to emerging methodologies for the elastic characterization of materials. Such developing methodologies are usually classified into approaches based on static tests and approaches based on dynamic tests.

The static approaches are generally based on the stresses calculated from direct measurement of strains undergone by suitable specimens, during certain mechanical tests (tensile, compression, bending, torsion etc.). ASTM [1] and ISO [2] provide many standards for determining the elastic properties of isotropic as well as composite materials. These norms recommend the use of standard sized and shaped specimens. In the case of composite materials, they involve the analysis of a large number of specimens and consequently tedious and time-consuming procedures [3].

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The above-mentioned methodologies usually require a significant range of stress-strain data to determine useful averaged values of the moduli. This necessarily involves destructive tests, as the deformation of the specimen must be measured until it fails, that is, until it deforms plastically or fractures. In either event, the sample is destroyed, and then is unavailable for further testing or other purposes.

In comparison with static approaches, dynamic approaches have the advantage of allowing the use of specimens with a greater variety of shapes and dimensions, and supplying, non-destructively, very precise measurements at a wide range of temperatures. Dynamic approaches can be classified into two groups: wave propagation based methods and modal vibration (or resonant) testing.

Much research has been dedicated to evaluating the possibility of measuring the material elastic properties by the methods belonging to the first group. Among these, the more commonly used is based on the measurement of the ultrasonic speed of wave propagation through the material or, in particular, the measurement of the transit time; i.e. the time that an ultrasonic impulse takes to cross a sample from the emitting transducer to the receiving transducer. Knowing the dimensions and the density of the specimen and the transit time of the transversal and longitudinal waves it is possible to calculate the Young's modulus and the shear modulus of the material. Although these techniques are robust and quick to perform, they suffer the disadvantage of being sensitive to possible local inhomogeneities of the material between the transducers [4].

Thick rectangular plates were used to determine all five of the engineering elastic constants of transversely isotropic materials. In this case, the transverse properties, such as transverse shear modulus, are determined by including not only the effects of bending, but also transverse shear and rotary inertia effects in describing the vibration behavior of the plates [5-12].

Ease of use and inexpensive equipment has recently increased the use of modal vibration testing in both research laboratories and industrial contexts. Such tests consist of making a specimen vibrate mechanically, at one or more vibration resonant modes. Knowledge of the resonant modal shapes and/or the values of the associated frequencies together with the sizes and mass of the sample allow the determination of the elastic constants of the material. ASTM had provided standardized procedures for testing isotropic materials [13-14].

II. IDENTIFICATION TECHNIQUE

Simulations have become an essential tool in the process that leads to the development of an engineering structure. Although these simulations have replaced a significant amount of experimental tests, they have not rendered testing obsolete. A successful simulation requires an accurate knowledge of the material parameters that are used in the numerical model. These parameters can only be obtained with an actual experiment. To improve the performance, durability or efficiency of mechanical equipment, material scientists are continuously developing new materials. Unfortunately, the mechanical behavior of these new materials is becoming increasingly more complex. The description of their behavior thus requires more elaborate models that use a larger number of model parameters. All these material parameters have to be measured in order to use the simulation models in a reliable way.

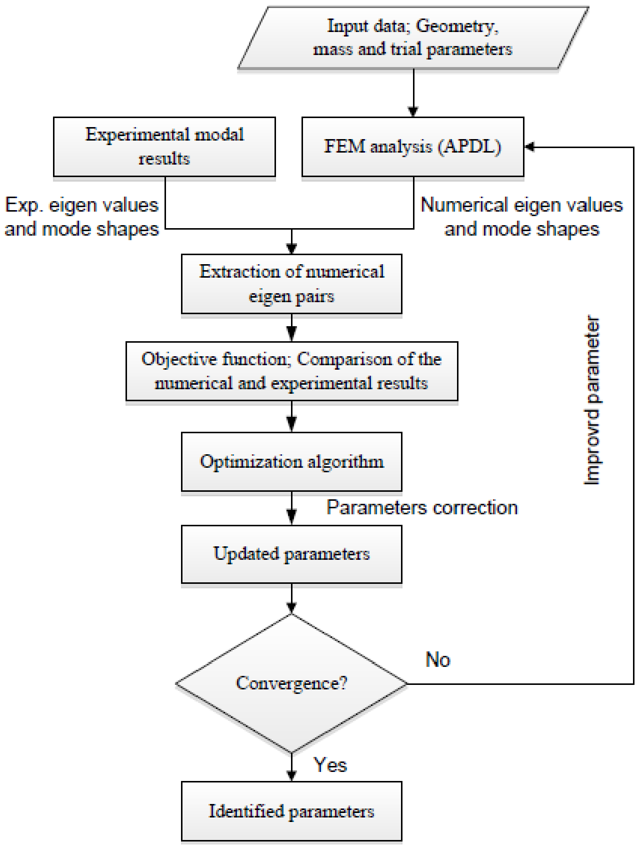

Figure1. Flow chart of mixed Numerical-experimental technique

However, the knowledge of these parameters is not only important for mechanical simulations. Since these material parameters represent the true behavior of the material, they can also be used by material scientists to evaluate and possibly improve the performance of newly developed materials. Metallic foam is one class of new materials that is becoming increasingly important for the production of high performance components. During design calculations, their stiffness properties are crucial for the assessment of stress fields. Numerous analysis techniques to identify the elastic properties of materials exist, however in the case of metallic foams, these techniques usually yield properties that are 'homogenised' over the material. This paper focuses on the development of identification procedures to identify the elastic parameters of aluminum foam.

In mixed numerical-experimental identification procedure, as shown in Figure 1, it is necessary to use an optimization method in order to identify the parameters of a numerical model, not invertible, to the target values that represent the experimental data. Thus, the optimization method is of prime importance in the parameter identification. However, the properties of error norms (or objective functions) to minimize also play an important role in the accuracy and robustness of the mixed numerical-experimental identification method, since these functions represent the extent of difference between numerical model of the current iteration and the experimental target values that the algorithm seeks to minimize. If these error norms are not sufficiently effective and does not capture all the physical effects of desired parameters, it is likely that a number of these parameters would ultimately find very poorly identified.

Thus in order to minimize the residual error in identification, while ensuring robustness and high rate of convergence, it is essential to develop a set of objective functions suitable for available experimental and numerical data that meet the following conditions.

- **Sensitivity to parameters**: the objective functions must be sufficiently sensitive to identify all the parameters to ensure an excellent accuracy of identification. In addition, a high sensitivity but balanced error norms reduces the effects on the identified parameters of measurement uncertainties and possible experimental errors.
- **Robustness**: the error norms used must be convex and having only a single local minimum over a wide range of parameters to ensure the smooth running of the optimization procedure regardless of the initial estimate of parameters to identify .
- **Precision**: the objective functions must converge towards zero near the actual parameters and the residual error in identification must not come from effects of numeric truncation or approximation in the calculation of these functions.

  The approach to develop the error functions and to develop the optimization algorithm is characterized by the following points:

1. Defining a set of functions possible based on the error norms commonly used in modal analysis and on original ideas.
2. Parametric study of large-scale FE on various types of aluminum foam plates to identify the robustness and sensitivity of the proposed error norms. The definition of the error norms function as a combination of selected elementary error functions.
3. Analysis, selection and implementation of a type of optimization algorithm corresponding to the needs.

III. ERROR FUNCTION

As commonly encountered in modal analysis, the most conventional modal error is obviously based on the relative difference of measured ω_k and calculated $\tilde{\omega}_k^i$ eigen frequencies for each mode k $(k = 1, 2, ..., m)$. However, it is important to identify that the experimental and the numerical mode shapes must absolutely match before making any comparison of frequencies. Also, a method to precisely define pairs of numerical mode and corresponding experimental must be applied before any calculations of modal error. The numerical and experimental meshes are not necessarily identical, these two modal vectors are not defined on the same grids. In addition, it should be noted that only normal components of the displacement of the specimen are measured whereas numeric modal vector contains all the components of the finite element model. The measured and calculated eigen modes are processed in a same reference mesh and only the degrees of freedom corresponding to the transverse displacements are retained.

The method of sorting and creation of pairs of modes used here is based on the MAC matrix of two sets of modes (in full), in which each column and each row correspond respectively to a mode of experimental model and a numerical model. For each mode of the experimental model the maximum component of the corresponding column of the MAC matrix is desired, thereby determining how numerical mode is best correlated with the experimental mode. If this component exceeds a maximum limit (classical tolerance 0.7), the mode pair is then validated.

It is therefore assumed here that the numerical and experimental modes index k actually forms a pair of corresponding modes. This modal error is implemented in the MATLAB program that calculates error vectors by comparing the experimental and numerical modal data. Table 1 shows the list of modal error norms used in proposed technique.

In order to avoid direct numerical packaging problems associated with highly variable magnitude of the parameters to identify (10^{10} difference between ν and E for example), the vector of parameters x^i is defined relative to the initial values. In summary, the problem of minimization with or without constraint can be defined as;

$$\underset{x^i \in \Re^n}{minimiser}\, f(x^i) \;\; with\; f(x^i) = \frac{1}{2}\|\mathrm{F}^{tot}(\mathrm{x}^i)\|^2 = \frac{1}{2}\sum_{k=1}^{q}[\mathrm{F}_k^{tot}(\mathrm{x}^i)]^2$$

With optional constraints

$$(x^{min})_j < (x^i)_j < (x^{max})_j$$

Where the error function can be written as;

$$\mathrm{F}^{tot} = \begin{bmatrix} \alpha^{frq}\mathrm{F}^{frq}, \alpha^{damp}\mathrm{F}^{damp}, \alpha^{mac1}\mathrm{F}^{mac1}, \alpha^{mac2}\mathrm{F}^{mac2}, \ldots \\ \ldots\, \alpha^{nod}\mathrm{F}^{nod}, \alpha^{ecm}\mathrm{F}^{ecm}, \alpha^{stab}(\mathrm{x}^i - \mathrm{x}^0) \end{bmatrix}^T$$

IV. MINIMIZATION ALGORITHMS

In an approach of mixed numerical-experimental technique, progressive convergence of numerical modal data of finite element model, whose parameters are unknown to the target values that represent the experimental data is usually performed iteratively with the aid of a minimization algorithm seeking to minimize the overall difference between these two data sets. In this technique the optimization algorithm plays an essential role since the convergence rate of the identification process and the resulting residual error depend directly on the effectiveness of the optimization step. In order to ensure the reliability of the identification method, the developments have been based on the well known Matlab optimization toolbox. By adopting a pragmatic development approach with regard to the identification algorithm, it is decided to base this work on an existing library of optimization routines that have already been proven in many applications. An important set of MATLAB routines for solving problems with parametric FE software ANSYS has been developed . As a part of minimization problem, the number n of parameters to identify (n = 3) and the number of components of error q (q = 6 × m, where m ≈ 10 is the number of measured modes) can be considered relatively low, thus classifying this problem in the category of small and medium-scale optimizations (low number of parameters, the small number of error norm not decoupled).

The main algorithms used in this field are either nonlinear least squares (Gauss-Newton, the maximum gradient or Levenberg-Marquardt methods) for unconstrained problems [15-16] or type sequential quadratic programming (SQP, Sequential Quadratic Programming) for constrained problems. Optimization by Nonlinear least squares method is used in the procedure for mixed numerical-experimental developed in this work.

In conclusion, the method of optimization by least squares Levenberg-Marquardt is recognized as being very robust and generally perform well in a wide variety of cases. Although theoretically converges slower than the Gauss-Newton in some very specific cases, this algorithm is most efficient on the majority of cases encountered. Moreover, the very rapid convergence of the algorithm, combined with the fact that it does not require systematic evaluation of gradients of error functions during the line search, this procedure is one of the least demanding in computing error norm and gradient.

V. MIXED NUMERICAL-EXPERIMENTAL TECHNIQUE

The approach of the proposed mixed numerical-experimental technique is;

a. import of the experimental modal model in MATLAB
b. initial estimation of the variable parameters
c. parametric FE model of test specimens
d. definition of the identification problem
e. execution of the optimization algorithm

VI. RESULTS AND DISCUSSION

Identification of variable parameters in parametric FE model is performed by choosing following initial values: Φ= 0.6, ν = 0.20 and damping = 5.0E-05. The formulation for calculation of Young's modulus and density pattern of the material is specified in the parametric FE model. The measured density array of 11000 components (computer tomography data) is given as input to the FE model of Testing structure, based on this density array 11000 materials are defined in this FE model. This measured density distributed model (MDDM) is used further for identification process. Automatic creation of numerical and experimental mode pairs based on the maximum terms of MAC, correlation matrix is used to ensure the compatibility of the eigen modes. Numerical mode shapes are interpolated on the same mesh size as that of experimental mesh (36×29). The mixed identification process is executed and after every optimization step each time a correlation is performed to choose the eigenvectors from the numerical eigen solution to be matched with the experimental ones, writing the Mode Pair Table (MPT).

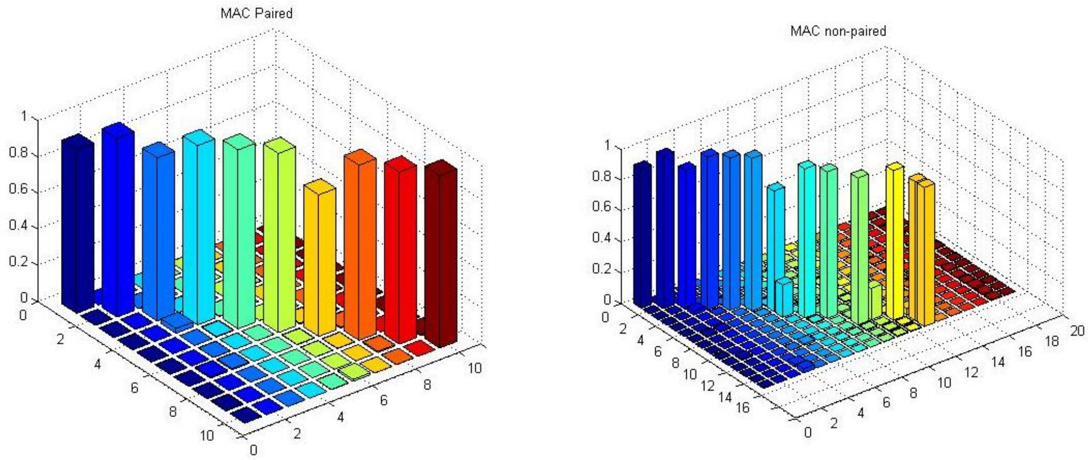

Figure 2 MAC matrix of experimental and numerical modes (paired and non-paired)

Due to change of the variable parameters, at each optimization step in the identification process, the numerical eigen modes related to the experimental ones might be shifted and interchanged in the sequence, which brings to a new evaluation of the MPT. An automatic routine has been developed to provide at each optimization step the updated MPT before executing the norm error function in the identification process. This step consists of computing the MAC values between the numerical mode shapes and the experimental mode shapes stored in the reference database. The combinations that result in the highest MAC-values are the mode shape pairs. The non-paired MAC matrix (modal correlation between all measured mode shapes and first 15 numerical mode shapes) and paired MAC matrix (modal correlation between first 10 experimental and corresponding numerical modes whose MAC values exceed 0.7) of Testing structure is shown in Figure 2. The first ten mode shape pairs having MAC values greater than 0.7 are further used in identification process.

The progress of the algorithm and evolution of variable parameters during the identification procedure are presented in Table 1. The convergence of the parameters is also presented graphically in Figure 3 and identified numerical eigen frequencies along with corresponding experimental eigen frequencies are illustrated in Table 2. The convergence of the Phi parameter and damping is fast while Poisson's ratio converges slightly slower and is less stable than others. The measured eigen frequencies and identified eigen frequencies for Testing structure at the end of optimization process are shown in Table 1. Quality of the identified mode shapes are excellent because the maximum error of the measured eigen frequencies and identified is ± 1.5 %. Diagonal terms of paired MAC matix shows the correlation between the first ten experimental and numerical modes, which is quite good as minimum component remains greater than 0.7, with an average of 0.9. The off diagonal terms of paired MAC matrix are very negligible which shows a good correlation between experimental and numerical mode shapes.

Table 1 Relative frequency error

Mode No.	1	2	3	4	5
Experimental Frequency (Hz)	219.85	281.33	462.62	525.82	597.41
Numerical Frequency (Hz)	220.75	279.21	463.95	523.31	601.07
Error (%)	0.41	-0.75	0.29	-0.48	0.61

Mode No.	6	7	8	9	10
Experimental Frequency (Hz)	797.26	993.60	1168.32	1313.01	1521.79
Numerical Frequency (Hz)	794.04	983.29	1185.00	1331.10	1502.30
Error (%)	-0.40	-1.04	1.43	1.38	-1.28

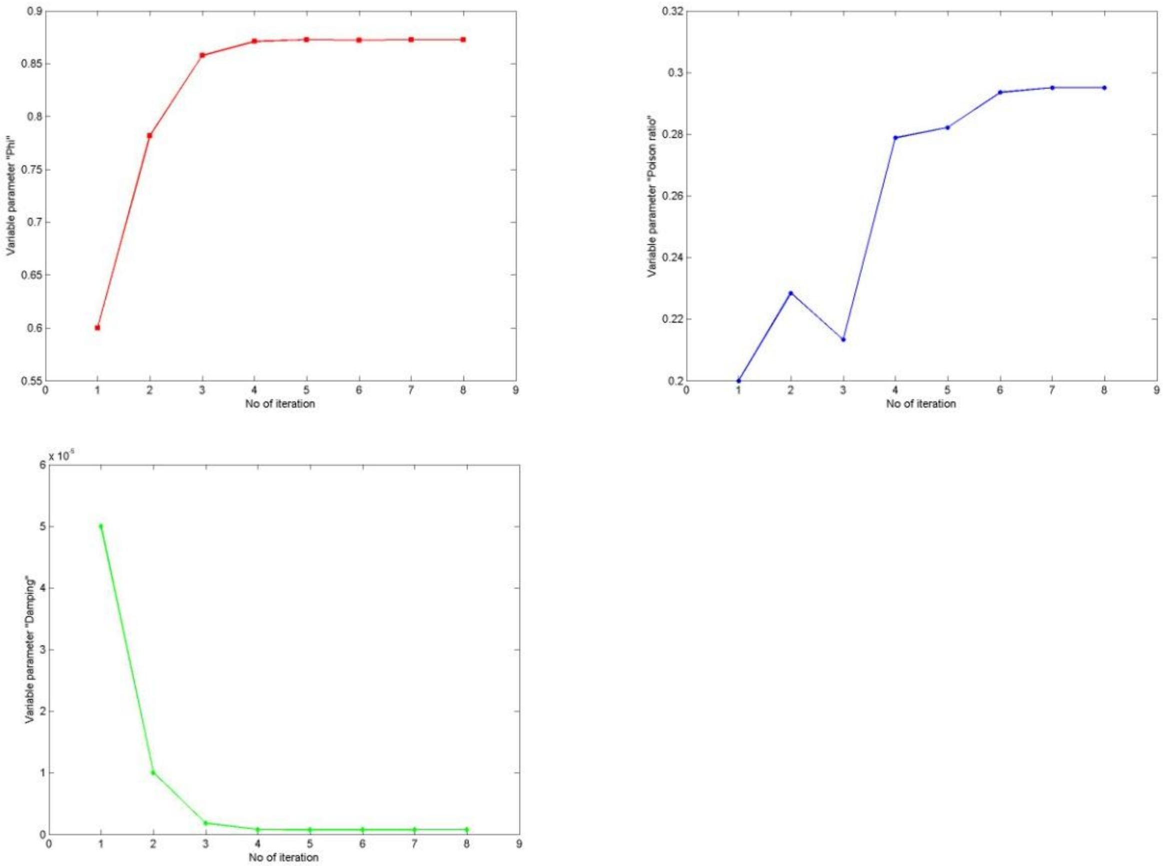

Figure 3 Graphs of convergence of identifying parameters

Table 2 Convergence of identifying parameters

PARAMETER	ITER.1	ITER.2	ITER.3	ITER.4	ITER.5	ITER. 6	ITER. 7	ITER. 8
PHI	0.6	0.782	0.858	0.871	0.873	0.872	0.873	0.873
POISSON'S RATIO	0.2	0.228	0.213	0.279	0.282	0.294	0.295	0.295
DAMPING	5.0E-5	1.0E-05	1.8E-06	7.9E-07	7.6E-07	7.6E-07	7.6E-07	7.6E-07

A mixed numerical-experimental identification technique which is based on experimental modal model, measured CT density distribution data and FE model is developed. To achieve this task an automation code in MATLAB was developed that incorporates ANSYS the finite element solver, experimental modal analysis results from LMS Test.Lab and a suitable optimization tool. A system call command in MATLAB was employed to call ANSYS for each simulation run that involved calculation of the modal responses to the input variable parameters. Modal results file of each FE simulation was saved for further process. Several MATLAB routines were developed to read these FE results files and extract the required data in matrix form. The experimental modal data was exported from LMS Test.Lab in a universal file format. This universal file was read by a MATLAB routine which extracts the required data in matrix form. A set of error functions based on measured and numerical eigen frequencies and corresponding mode shapes was developed in MATLAB. These error functions have been combined to form a total functional error for optimization of the FE model input variable parameters. Levenberg-Marquardt algorithm with assessment of gradients by finite difference method was used for optimization. Proposed optimization method has proved very effective, since on average, all variable parameters converged at 5th iteration.

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REFERENCES

1. *American Society for Testing and Materials, ASTM International Standards Worldwide, 100 Barr Harbor Drive, West Conshohocken, Pennsylvania, USA*. Available from: http://www.astm.org/.
2. *International Organization for Standardization, ISO, 1, ch. de la Voie-Creuse, DCase postale 56DCH-1211 Geneva 20, Switzerland*. Available from: http://www.iso.org/.
3. Pipes, L., L.A. Carlsson, and D.F. Adams, *Experimental Characterization of Advanced Composite Materials*. 1997, Technomic Publishing Co. Inc.
4. Pagnotta, L., *Recent progress in identification methods for the elastic characterization of materials.* International Journal of Mechanics, 2008. **2**(4): p. 129-140.
5. Maletta, C. and L. Pagnotta, *On the determination of mechanical properties of composite laminates using genetic algorithms.* International Journal of Mechanics and Materials in Design, 2004. **1**(2): p. 199-211.
6. Hwang, S.-F. and C.-S. Chang, *Determination of elastic constants of materials by vibration testing.* Composite Structures, 2000. **49**(2): p. 183-190.
7. Rikards, R., et al., *Method for identification of elastic properties of laminates based on experiment design.* Composites Part B: Engineering, 1999. **30**(3): p. 279-289.
8. Silva, M.F.T., et al., *A genetic algorithm applied to composite elastic parameters identification.* Inverse Problems in Science and Engineering, 2004. **12**(1): p. 17-28.
9. Wereszczak, A., R.H. Kraft, and J.J. Swab. *Flexural and torsional resonances of ceramic tiles via impulse excitation of vibration*. 2003: Wiley Online Library.
10. Ayorinde, E.O., *Elastic constants of thick orthotropic composite plates.* Journal of Composite Materials, 1995. **29**(8): p. 1025.
11. Gagneja, S., R.F. Gibson, and E.O. Ayorinde, *Design of test specimens for the determination of elastic through-thickness shear properties of thick composites from measured modal vibration frequencies.* Composites Science and Technology, 2001. **61**(5): p. 679-687.
12. Hua, H., *Identification of plate rigidities of anisotropic plate rectangular plates, sandwich panels and circular orthotropic discs using vibration data*, PhD dissertation, Free University of Brussels, 1993.
13. *ASTM Standard E1876-07, Standard test method for dynamic Young's modulus, shear modulus, and Poisson's ratio by impulse excitation of vibration*, in *Book of Standards, Volume 03.01*.
14. *ASTM Standard E1875-08, Standard test method for dynamic Young's modulus, shear modulus, and Poisson's ratio by sonic resonance* in *Book of Standards, Volume 03.01*.
15. Madsen, K., H. Bruun, and O. Tingleff, *Methods for non-linear least squares problems.* 1999.
16. More, J., *The Levenberg-Marquardt algorithm: implementation and theory*. Numerical analysis, 1978: p. 105-116.