Analytical Design of State Feedback Controllers for a Nonlinear Interacting Tank Process

S. Nagammai, S.Latha, I.Aarifa, S.D.Idhaya

***Abstract:** The typical nonlinear interacting tank process has the difficulty in controller design because of a change in system dynamics and interaction of processes. This paper deals with design methodology of state feedback with and without integral controller and the performance of which is compared with ZN tuned PID controller. A simulation is carried out using MATLAB to control the modeled nonlinear interacting tank processes.*

***Keywords:** Interacting tank process, PID controller, state feedback controller.*

I. INTRODUCTION

The control of liquid level is a basic problem in the process industries such as Petrochemical industries, paper making process or mixing treatment in the tanks. It is essential for control system engineers to understand how the level control problem is solved. The problem of level control in a tank process with variable area is very cumbersome. Many control methods such as Soft Computing techniques [3], fuzzy controller [4] have been applied to control level in a spherical tank process.

The paper is organized as follows. Section II gives details about the mathematical modeling of nonlinear interacting tank processes. In Section III the state space modeling of the plant is discussed. In Section IV the design of PID controller is discussed. In Section V the design of state feedback controller concepts are discussed. Section VI explains an implementation of state feedback with integral controller for interacting tank process. The simulated results are given in section VII. The final conclusion is given in section VIII.

II. PROCESS DESCRIPTION

A. Test example1: Interacting spherical tank process

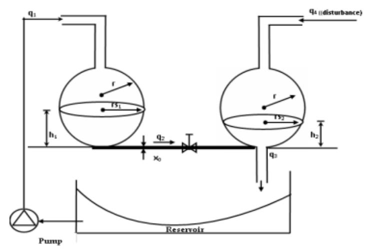

Fig.1. Schematic diagram of nonlinear interacting Spherical tank Process

Manuscript received on April, 2013.

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Consider an interacting spherical tank process, with one input & one output as shown in Fig 1. The objective is to control the water level of tank 2 namely $h_2(t)$ by the inlet water flow $q_1(t)$. Here $q_4(t)$ assumed as a disturbance variable. The control objective is to maintain a level in tank 2 by the varying inflow rate of tank1 in presence of disturbance flow to tank 2.Using the law of conservation of mass, the nonlinear plant equations are obtained.

Mathematical modeling of interacting spherical tank process:

For Tank1 $q_1 - q_2 = A_1 \dot{h}_1$

For Tank2 $q_4 + q_2 - q_3 = A_2 \dot{h}_2$ (1)

Where

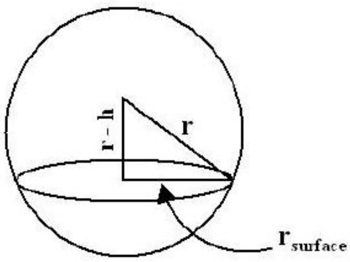
q_1 --- In flow rate to tank 1 (cm^3 / sec)

q_3 --- Out flow rate of tank2 (cm^3 / sec)

q_2 --- Flow between the tanks (cm^3 / sec)

q_4 --- Disturbance flow to tank2 (cm^3 / sec)

The radius on the surface of the fluid varies according to the level (height) of fluid in the tank, and let this radius be known as $r_{surface}$.



$$length^2 + height^2 = hypotenuse^2$$

Let,

Length= $r_{surface}$

Height=radius of tank(r)-fluid level (h_1)

Hypotenuse=radius of tank(r)

$r_{surface}$ = radius at any level

Table. 1. Interacting Spherical tank process parameters and Steady state values:

variable	Description	Values
r	radius of tank (m) (r_1=r_2)	0.75
g	gravitational force (m^2/sec)	9.81
x_0	thickness(diameter)of pipe (m)	0.06
h_{1s}	steady state water level of tank 1 (m)	0.3
h_{2s}	steady state water level of tank 2 (m)	0.1
a	area of connecting pipe (m^2)	0.0028
A_1	area of tank1 (m^2)	1.131
A_2	area of tank2 (m^2)	0.44

$$r_{surface1} = \sqrt{2r_1h_1 - h_1^2}$$

$$r_{surface2} = \sqrt{2r_2h_2 - h_2^2}$$

Hence, $A(h) = \pi\left(2rh_s - h_s{}^2\right)$

Also

$$q_2 = a\sqrt{2g(h_1 - h_2)}$$

$$q_3 = a\sqrt{2g(h_2 - x_0)}$$

Where $a = \pi\left(\frac{x_0}{2}\right)^2$

By means of Taylor series approximation the nonlinear model equations given in equation (1) are linearized and the state space model thus obtained is given by equation (2).

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -0.011 & 0.011 \\ 0.03 & -0.1 \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 0.884 \\ 0 \end{bmatrix}[q_1] + \begin{bmatrix} 0 \\ 2.27 \end{bmatrix}[q_4]$$

$$[y_1] = [0 \quad 1]\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \qquad (2)$$

B. Test example 2: Interacting three tank process

Consider an interacting cylindrical three tank process, with one input & one output as shown in Fig 3. The control objective is to maintain a level in tank 3 by the varying inflow rate of tank1 in presence of disturbance flow to tank 3. Here $q_4(t)$ is assumed as a disturbance variable.

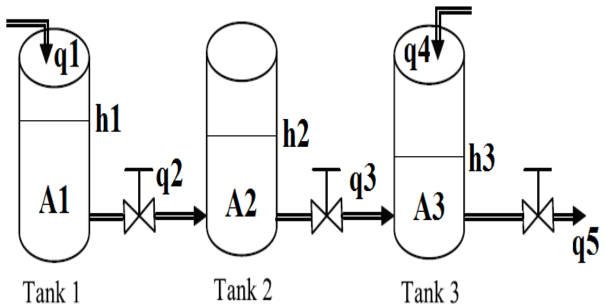


Fig-3: schematic diagram of interacting three tank processes

Using the law of conservation of mass, the nonlinear plant equations are obtained.

Mathematical modeling of interacting three tank processes:

For Tank 1, $q_1 - q_2 = A_1 \dot{h}_1$

For Tank 2, $q_2 - q_3 = A_2 \dot{h}_2$

For Tank 3, $q_3 + q_4 - q_5 = A_3 \dot{h}_3 \qquad (3)$

Where, $q_2 = ac_1\sqrt{2g(h_1 - h_2)}$

$$q_3 = ac_2\sqrt{2g(h_2 - h_3)}$$

$$q_5 = ac_3\sqrt{2g(h_3)}$$

Table. 2. Interacting three tank process parameters and Steady state values:

variable	Description	Values
g	gravitational force(m^2/sec)	9.81
a	area of pipe (m^2)	$5X10^{-5}$
h_{1s}	steady state water level of tank 1 (m)	0.5
h_{2s}	steady state water level of tank 2 (m)	0.45
h_{3s}	steady state water level of tank 2 (m)	0.4
A_1,A_2,A_3	area of tank1 (m^2)	0.0154
C_1	Valve coefficient of tank1	1
C_2	Valve coefficient of tank2	0.8
C_3	Valve coefficient of tank3	1

By means of Taylor series approximation the nonlinear model equations given in equation (3) are linearized and the state space model thus obtained is given by equation (4).

$$\begin{pmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{pmatrix} = \begin{bmatrix} -0.0322 & 0.0322 & 0 \\ 0.0322 & -0.0578 & 0.0257 \\ 0 & 0.0257 & -0.0371 \end{bmatrix}\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} + \begin{pmatrix} 64.93 \\ 0 \\ 0 \end{pmatrix}u_1 + \begin{pmatrix} 0 \\ 0 \\ 64.93 \end{pmatrix}u_2$$

$$y = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \qquad (4)$$

In this control design problem, the linearized plant model relating level in tank 3 to inflow rate to tank1 and the disturbance model is derived as given by equation (5).

$$\frac{H_3(s)}{Q_1(s)} = \frac{0.054}{s^3 + 0.1272s^2 + 0.0035s + 9.4x10^{-6}}$$

And the disturbance model is,

$$\frac{H_3(s)}{Q_4(s)} = \frac{64.93s^2 + 5.885s + 0.0538}{s^3 + 0.1272s^2 + 0.0035s + 9.4x10^{-6}} \qquad (5)$$

III. STATE SPACE MODELING IN CONTINUOUS DOMAIN

A. Test example1: Interacting spherical tank process:

Based on the plant transfer function given as in Equation 3, the state variables of the interacting spherical tank system are defined as:

$x_1(t) = h_2(t) =$ Water level of tank2

$x_2(t) = \frac{dh_2(t)}{dt} =$ Rate of change of water level in tank2.Meanwhile, the state input and state output for the second order system are defined as:

$u(t) = q_1(t)$ = in flow rate to tank1

$y(t) = h_2(t)$ = water level of tank2

$$\dot{x}_1(t) = \dot{h}_2(t) = x_2(t)$$

$$\dot{x}_2(t) = \ddot{h}_2(t)$$

From, Equation (3) it is obtained that,

$$\dot{x}_2(t) = -0.0008x_1(t) - 0.111x_2(t) + 0.0266u(t)$$

Therefore, the state space representation of the Interacting spherical tank process is expressed as given in equation (6)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.0008 & -0.111 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0266 \end{bmatrix} u$$

$$\text{and } y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (6)$$

B. Test example 2: Interacting three tank process

Based on the plant transfer function given as in Equation 5, the state variables of the spherical interacting tank system are defined as:

$x_1(t) = h_3(t) =$ Water level of tank3

$x_2(t) = \frac{dh_3(t)}{dt} =$ Rate of change of water level in tank3

$$x_3(t) = \frac{d^2h_3(t)}{dt^2}$$

The state input and state output for the system are defined as,

$u(t) = q_1(t)$ = in flow rate to tank1

$y(t) = h_3(t)$ = water level of tank3

Let

$$x_1(t) = h_3(t)$$

$$\dot{x}_1(t) = x_2(t) = \dot{h}_3(t)$$

$$x_3(t) = \dot{x}_2(t) = \ddot{h}_3(t)$$

$$\dot{x}_3(t) = \dddot{h}_3(t)$$

From, Equation (5) it is obtained that,

$$\dot{x}_3(t) = -0.0000094x_1(t) - 0.0035x_2(t) - 0.127x_3(t) + 0.054u(t)$$

Therefore, the state space representation of the interacting three tank process is expressed as given in equation[7]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.0000094 & -0.0035 & -0.127 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.054 \end{bmatrix} u$$

$$\text{and } y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (7)$$

IV. PID CONTROLLER DESIGN

A. Test example1: Interacting spherical tank process

The FOPDT model of interacting spherical tank process is obtained using two point methods and is given by equation [8]

$$G(s) = \frac{k_p e^{-t_d s}}{\tau_p s + 1} = \frac{33.25e^{-10.4s}}{129s + 1} \qquad (8)$$

The value of the frequency of self sustained oscillation is obtained as $\omega_{c0} = 0.155 rad/\sec$

$$k_{cu} = 0.06, \; P_u = \frac{2\pi}{\omega_{c0}} = 40.53$$

The ZN-PID tuning parameters are,

$$K_c = 0.36, \; K_i = \frac{K_c}{\tau_i} = 0.018, K_d = K_c * \tau_d = 1.82$$

B. Test example 2: Interacting three tank process:

The FOPDT model of interacting three tank process is obtained using two point methods and is given by equation [9]

$$G(s) = \frac{k_p e^{-t_d s}}{\tau_p s + 1} = \frac{5744e^{-48.56s}}{328.6s + 1} \qquad (9)$$

The value of the frequency of self sustained oscillation is obtained as $\omega_{c0} = 0.034 rad/\sec$

$$k_{cu} = 0.00195, \; P_u = \frac{2\pi}{\omega_{c0}} = 0.034$$

The ZN-PID tuning parameters are,

$$K_c = 0.0012, \; K_i = \frac{K_c}{\tau_i} = 0.000013, K_d = K_c * \tau_d = 0.0277$$

V. STATE FEEDBACK CONTROLLER DESIGN

The concept of feed-backing all the state variables back to the input of the system through a suitable feedback gain matrix in the control strategy is known as the full-state variable feedback control technique. The structure of state feedback controller is shown in Fig.4.a. This design technique starts with the determination of desired closed-loop poles to satisfy transient response requirements. By choosing an appropriate gain matrix, K for state feedback, it is possible to force the system to have closed loop poles at the desired locations, provided that the original system is completely state controllable. In this design technique it is assumed that all state value variables are measurable and are available for feedback. Normally, the major disadvantage of the design of the state feedback controller using pole-placement is large offset. Therefore, in order to compensate for this problem, an integral control is added which eliminates the offset in the response to the step input. The structure of state feedback controller with integral action is shown in Fig.4.b.

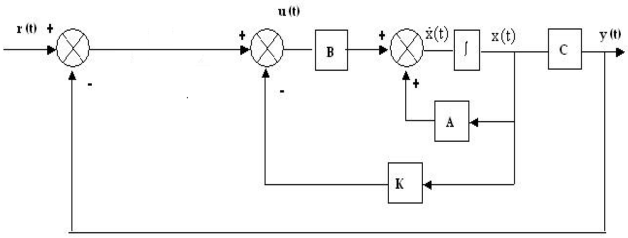


Fig.4.a. Block diagram of state feedback controller with plant

VI. STATE FEEDBACK WITH INTEGRAL CONTROL DESIGN

Fig.4.b shows the block diagram of a state feedback with an integral control system that is composed of plant, state feedback controller and an Integrator. A feedback path from the output has been added to the error, e, which is fed forward to the controller via an Integrator. The main function of adding an Integrator is to increase the speed of system response thus reduces the finite steady-state error.

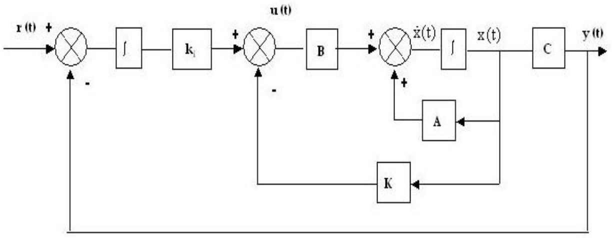


Fig.4.b: Block diagram of state feedback with integral control.

One way to introduce integral control is to augment the state vector with the desired integral. More specifically, the state 'x' as well as the integral of the error is fed back by augmenting the plant state 'x' with the extra 'integral state' z defined by the equation,

$$\dot{z}(t) = r - y(t) = r - Cx(t)$$

Hence the system matrix with integral control is given as [10]

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}\begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \qquad (10)$$

An asymptotically stable solution to this problem exists, and is given by,

Table 3: State feedback gain values:

Controller Type	Interacting spherical Tank process			Interacting three Tank process			
State feedback Controller	k_1	k_2		k_1	k_2	k_3	
	0.45	1.84		11	38.5	36.5	
State feedback with integral Controller	k_1	k_2	k_i	k_1	k_2	k_3	k_i
	3.46	20.6	-0.24	23.4	50	42.1	-3.6

VII. SIMULATION RESULTS

In order to analyze the performances of the proposed controller, the system is simulated using MATLAB/SIMULINK. The open loop response of the designed system is shown in Fig.5.a and Fig.5.b.and which indicates that, the open loop system is stable but set point tracking is not obtainable. To obtain output response with 4% overshoot, settling time of 60 seconds the desired pole locations are assumed suitably. The servo response of ZN tuned PID controller in comparison with state feedback controller with and without integral action is shown in Fig 6.a and Fig.6.b for both the designed processes for continuous step change in set point. In order to demonstrate the disturbance rejection capability of various control schemes, simulation studies have been carried by changing in flow to another tank as disturbance variable and which is shown in Fig.7.a and Fig.7.b.

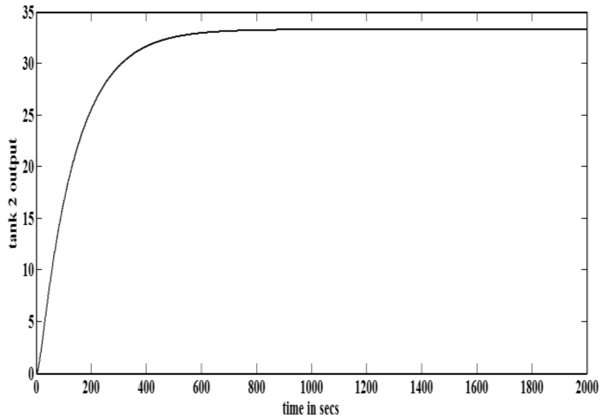


Fig.5.a.Open loop response of interacting spherical tank process for set point change in inflow to tank1

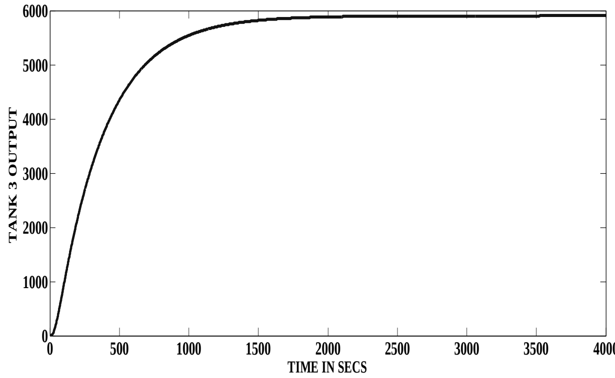


Fig.5.b.Open loop response of interacting three tank process for set point change in inflow to tank1

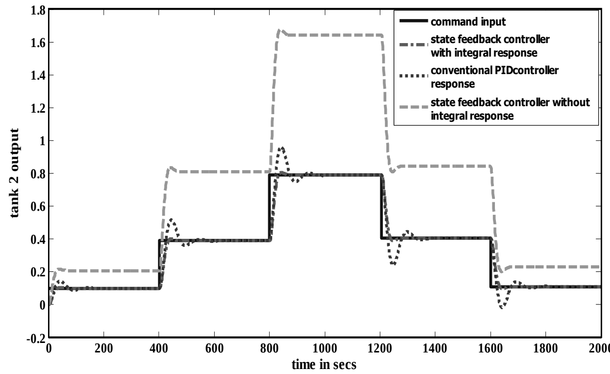


Fig.6.a.Servo response of interacting spherical tank process for set point change in Inflow to tank1

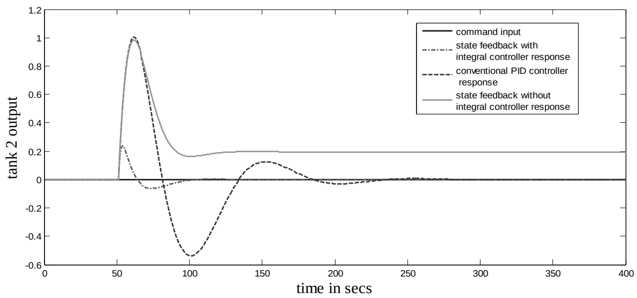


Fig.6.b.Regulatoryresponse of interacting spherical tank process for step point change in inflow to tank2 at t=50 sec

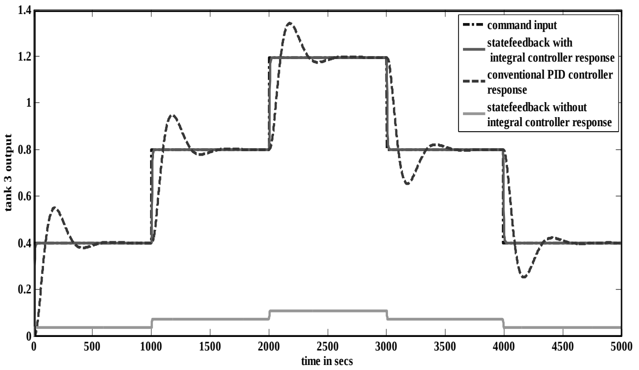


Fig.7.a.Servo response of interacting three tank processes for set point change in Inflow to tank1

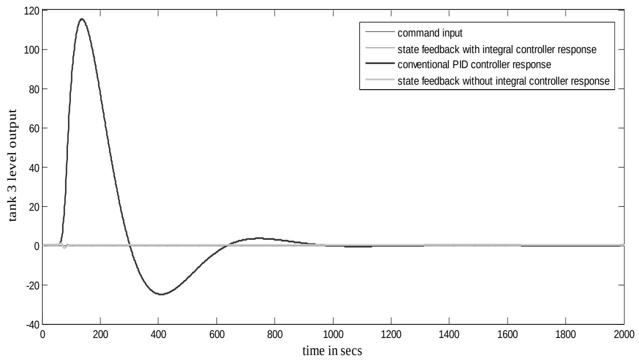


Fig.7.b.Regulatory response of interacting three tank process for set point change in Inflow to tank3 at t=50sec

VIII. CONCLUSION

The proposed algorithm is demonstrated for a nonlinear interacting tank process. It has been shown that, the state feedback controller tracks the set point with minimum settling time. The state feedback controller offers less overshoot than PID controller. The performance summary given in Table 4 and Table 5 indicates that, the actual time domain specifications are close to the desired specifications for state feedback controller with integral action than with other two control schemes. One of the drawback of the state feedback controller without integral action is it exhibits offset.

Table 4: Comparison of Time domain Specifications and Performance indices for interacting spherical tank Process

Parameter	State feed back with integral Control	Traditional PID Control	State feedback without Integral control
Peak time in secs	43.2	40.42	40.43
Rise time in secs	18.4	12.52	17.32
Settling time in secs	61.61	158.82	56.74
$\%M_P$	4.04	43.32	3.79
IAE	11.18	14.24	8.2
ISE	16.23	30.37	39.09

Table 5: Comparison of Time domain Specifications and Performance indices for Interacting three tank Process

Parameter	State feedback with integral Control	Traditional PID Control	State feedback without Integral control
Peak time in secs	856	178	11
Rise time in secs	7	63	4
Settling time in secs	21	544	14
%Mp	0.0051	37.53	2.18
IAE	461.5	575.2	1,365
ISE	418	474.6	1,241

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