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Bit Error Rate Performance of BPSK Modulation and OFDM-BPSK with Rayleigh Multipath Channel

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Abstract---In this paper Bit Error Rate performance of BPSK modulation and OFDM -BPSK System over Rayleigh fading channel is analyzed. The performance of BER of BPSK over AWGN and Rayleigh channel is compared. OFDM is a orthogonal frequency division multiplexing to reduce inter-symbol interference problem. Simulation of BPSK signals is carried with both AWGN and Rayleigh channel. Finally simulations of OFDM signals are carried with Rayleigh faded signals to understand the effect of channel fading and to obtain optimum value of Bit Error Rate (BER) and Signal to noise ratio (SNR). The simulation results show that the simulated bit error rate is in good agreement with the theoretical bit error rate for BPSK modulation.

Keywords-BPSK, BER, OFDM, Rayleigh fading channel etc.

I. INTRODUCTION

A very popular digital modulation scheme, binary phase shift keying (BPSK) shifts the carrier sine wave 180° for each change in binary state*)*. BPSK is coherent as the phase transitions occur at the zero crossing points. The proper demodulation of BPSK requires the signal to be compared to a sine carrier of the same phase. Orthogonal Frequency Division Multiplexing [1] (OFDM) has been adopted as the modulation technique for various current and proposed wireless systems. It provides high data rates, high spectral efficiency. In this paper we will compare the bit error rate performance of BPSK modulation and OFDM-BPSK with Rayleigh multipath channel.

II. BER for BPSK MODULATION:

BPSK is the simplest form of phase shift keying (PSK). In BPSK, individual data bits are used to control the phase of the carrier. During each bit interval, the modulator shifts the carrier to one of two possible phases, which are 180 degrees or π radians apart as shown in Fig.1. The theoretical equation for bit error rate (BER) with Binary Phase Shift Keying (BPSK) modulation scheme in Additive White Gaussian Noise (AWGN) channel will be derived.

The AWGN channel is a good model for many satellite and deep space communication links. It is not a good model for most terrestrial links because of multipath, terrain blocking, interference, etc. However, for terrestrial path modeling, AWGN is commonly used to simulate background noise of the channel under study, in addition to multipath, terrain blocking, interference, ground clutter and self interference that modern radio systems encounter in terrestrial operation.

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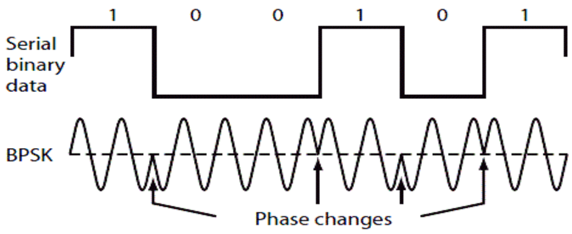

Figure1: In Binary phase sift keying a binary 0 is 0^0 while a binary 1 is 180^0

The block diagram of BPSK transmitter- receiver[2] is as shown in Fig.2, and With Binary Phase Shift Keying (BPSK), the binary digits 1 and 0 maybe represented by the analog levels $+\sqrt{E_b}$ and $-\sqrt{E_b}$ respectively is as shown in the Fig.2.

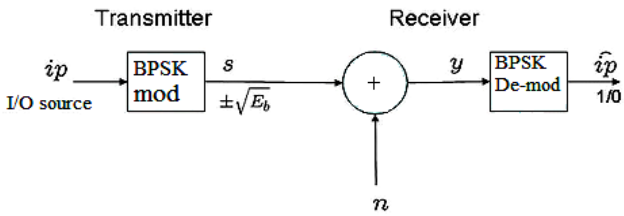

Figure 2. Block Diagram of BPSK transmitter- receiver

As the noise, n, gets added to the received signal, the value of noise follows the Gaussian probability distribution function given by,

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(n-\mu)^2}{2\sigma^2}} \quad (1)$$

with μ=0 and $\sigma^2 = \frac{N_0}{2}$

The bit error probability is given by,

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \quad (2)$$

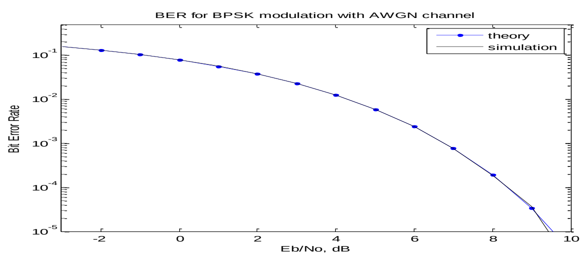

Figure 3: BER curve for BPSK modulation.

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A. Rayleigh Fading Model:

In a multipath environment, it is reasonably intuitive to visualize that an impulse transmitted from transmitter will reach the receiver as a train of impulses. The Multipath Rayleigh Fading Channel implements a baseband simulation of a multipath Rayleigh fading propagation channel[3]-[4]. This channel is useful for modeling mobile wireless communication systems.

Relative motion between the transmitter and receiver causes Doppler shifts in the signal frequency. The Jakes PSD (power spectral density) determines the spectrum of the Rayleigh process. Since a multipath channel reflects signals at multiple places, a transmitted signal travels to the receiver along several paths that may have different lengths and hence different associated time delays. Fading occurs when signals traveling along different paths interfere with each other.

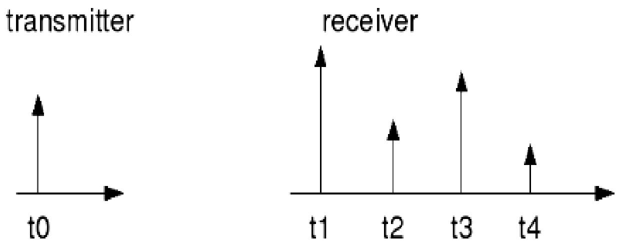

Figure 4: Impulse response of a multipath channel

Let the transmit band pass signal be

$$x(t) \;=\; R\{x_b(t)e^{j2\pi f_c t}\} \qquad (3)$$

Where

$x_b(t)$ is the base band signal
f_c is the carrier frequency
t is the time

As shown above, the transmit signal reaches the receiver through multiple paths where the nth path has an attenuation $\alpha_n(t)$ and delay $\Gamma_n(t)$. The received signal is,

$$r(t) = \sum_n \alpha_n(t)x[t - \Gamma_n(t)] \qquad (4)$$

Plugging in the equation for transmit baseband signal from the above equation,

$$r(t) = R\left\{\sum_n \alpha_n(t)x_b[t-\Gamma_n(t)]e^{j2\pi f_c[t-\Gamma_n(t)]}\right\} \qquad (5)$$

The baseband equivalent of the received signal is,

$$r_b(t) = \sum_n \alpha_n(t)e^{-j2\pi f_c \Gamma_n(t)}\, x_b[t-\Gamma_n(t)]$$

$$= \sum_n \alpha_n(t)e^{-j\theta_n(t)}\, x_b[t-\Gamma_n(t)] \qquad (6)$$

Where $\theta_n(t) = 2\pi f_c \Gamma_n(t)$ is the phase of the nth path. The impulse response is,

$$h_b(t) = \sum_n \alpha_n(t)e^{-j\theta_n(t)} \qquad (7)$$

A circularly symmetric complex random variable **Z,** which is a combination of real and imaginary parts,

$$E[Z] = E[e^{j\theta}Z] = e^{j\theta}E[Z] \qquad (8)$$

A circularly symmetric complex Gaussian random variable is completely specified by the variance,

$$\sigma^2 = E[Z^2] \qquad (9)$$

The Rayleigh random variable which is nothing but probability density function is given by,

$$p(Z) = \frac{z}{\sigma^2}e^{\frac{-z^2}{2\sigma^2}}\;, z\geq 0 \qquad (10)$$

In order to get the bit error rate for BPSK with Rayleigh fading channel which is of the form y= zx+n, assume that the channel is flat fading and is randomly varying in time. The noise, n has the Gaussian probability density function which is given by equation (1) i.e; $p(n) = \frac{1}{\sqrt{2\sigma^2}}e^{\frac{-(n-\mu)^2}{2\sigma^2}}$ Equation (2) gives the Bit error probability, however the presence of channel z, the effective bit energy to noise ratio is $\frac{|z|^2E_b}{N_0}$. so the bit error rate probability for a given value of z is given by,

$$P_{b|z} = \frac{1}{2}erfc\left(\sqrt{\frac{|z|^2E_b}{N_0}}\right) \;=\; \frac{1}{2}erfc(\sqrt{\gamma}) \qquad (11)$$

Where $\gamma = \frac{|z|^2E_b}{N_0}$. So the error probability is given by,

$$P_b = \frac{1}{2}\left(1-\sqrt{\frac{\left(E_b/N_0\right)}{\left(E_b/N_0\right)+1}}\right) \qquad (12)$$

From above equation we can draw the bit error rate curve for BPSK using Rayleigh channel.

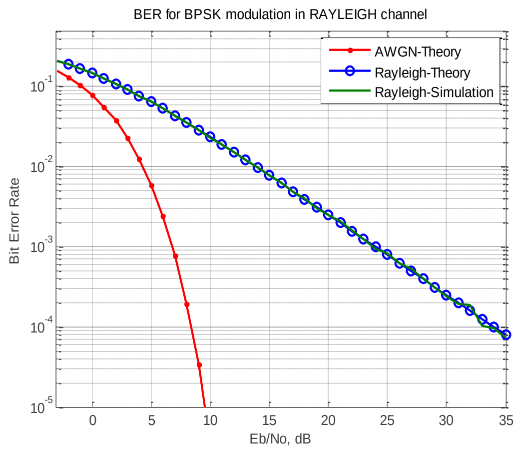

Figure 5: Comparision between AWGN and Rayleigh channel BER for BPSK

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III. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM)

Orthogonal frequency division multiplexing (OFDM) combines modulation and multiplexing techniques to improve spectral efficiency. A transmission channel is divided into many smaller sub channels or subcarriers. The subcarrier frequencies and spacing are chosen so they are orthogonal to one another. Their spectra won't interfere with one another, then, so no guard bands are required.

Most broadband systems are subject to multipath transmission. Conventional solution to multipath is equalizer in the receiver but for high data rates equalizers are too complicated. With OFDM there is a simple way of dealing with multipath which makes use of relatively simple DSP algorithms. OFDM solves the problem of multipath by transmitting the data in parallel with longer symbol period and by cyclic prefix to reduce Inter Symbol Interference [5]. In OFDM parallel data streams are used as inputs to an IFFT. IFFT output is sum of signal samples. IFFT does modulation and multiplexing in one step. Filtering and D/A of samples results in baseband signal as shown in Fig. 6

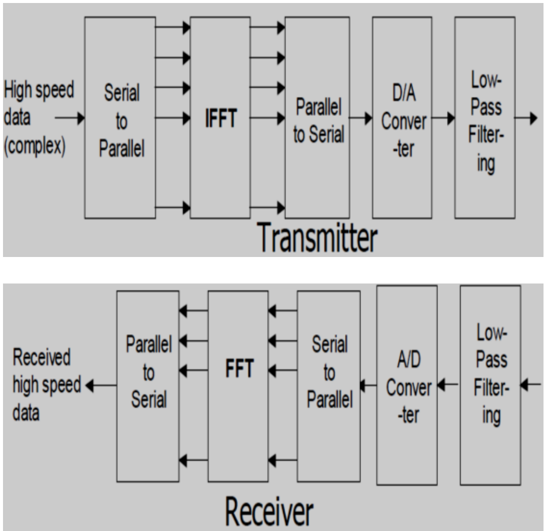

Figure 6: Baseband OFDM system.

A. BER of OFDM-BPSK:

The total channel is a frequency selective channel [3]-[8], the channel experienced by each subcarrier in an OFDM system is a flat fading channel with each subcarrier experiencing independent Rayleigh fading. So, assuming that the number of taps in the channel is lower than the cyclic prefix as shown in Figure. 7 duration (which ensures that there is no inter symbol interference), the BER for BPSK with OFDM in a Rayleigh fading channel should be same as the result obtained for BER for BPSK in Rayleigh fading channel.

To eliminate ICI, the OFDM symbol is cyclically extended in the guard interval. The cyclic prefix is actually a copy of the last portion of the data symbol appended to the front of the symbol during the guard interval.

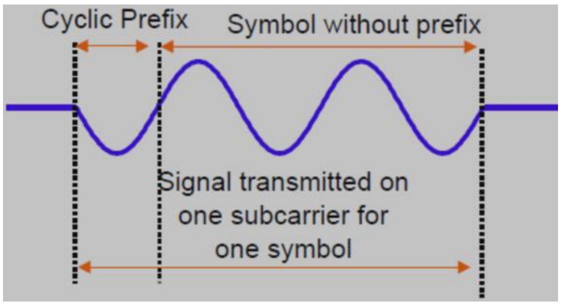

Figure 7: Cyclic prefix

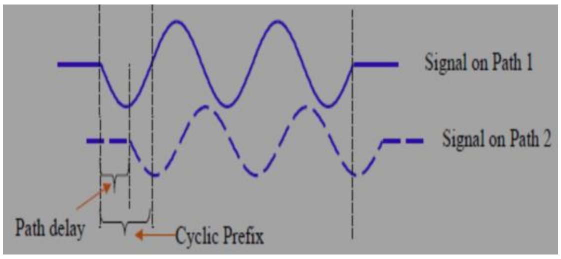

Figure 8: Effect of multipath on symbol with cyclic prefix

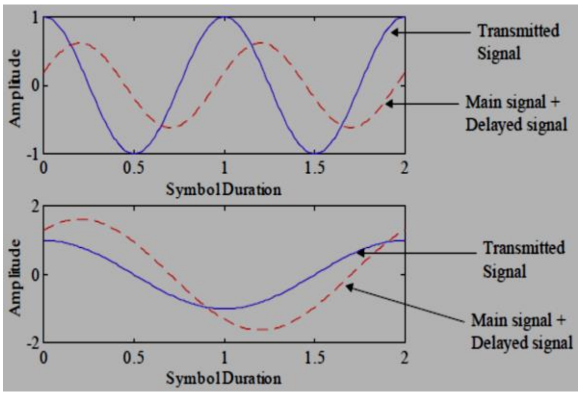

Figure 9: Frequency selective fading

Let us use an OFDM system based on IEEE 802.11a specifications which are shown in Table. 1.

Parameter	Value
FFT size. nFFT	64
Number of used subcarriers. nDSC	52
FFT Sampling frequency	20MHz
Subcarrier spacing	312.5kHz
Used subcarrier index	{-26 to -1, +1 to +26}
Cyclic prefix duration, Tcp	0.8us
Data symbol duration, Td	3.2us
Total Symbol duration, Ts	4us

Table 1: specifications of OFDM system

The relation between symbol energy and the bit energy is given by,

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$$\frac{E_s}{N_0} = \frac{E_b}{N_0}\left(\frac{nDSC}{nFFT}\right)\left(\frac{T_d}{T_d + T_{cp}}\right) \qquad (12)$$

Expressing in decibels,

$$\frac{E_s}{N_0}dB = \frac{E_b}{N_0}dB + 10log_{10}\left(\frac{nDSC}{nFFT}\right) + 10log_{10}\left(\frac{T_d}{T_d+T_{cp}}\right) \qquad (13)$$

The bit error rate for BPSK with Rayleigh channel [9]-[10] is same as that of Equation (12). Simulation results for bit error rate of BPSK with OFDM with AWGN and Rayleigh channel are as shown in figure.10and figure.11respectively.

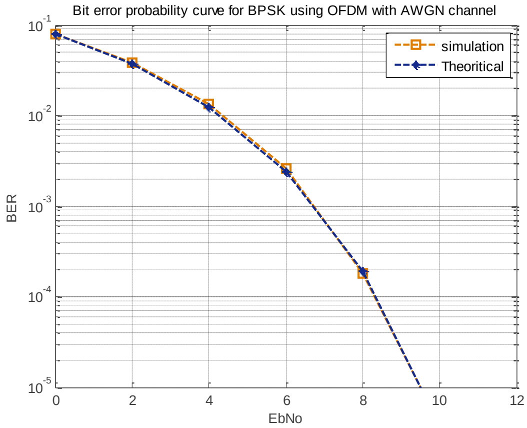

Figure 10: BER curve for OFDM-BPSK with AWGN channel

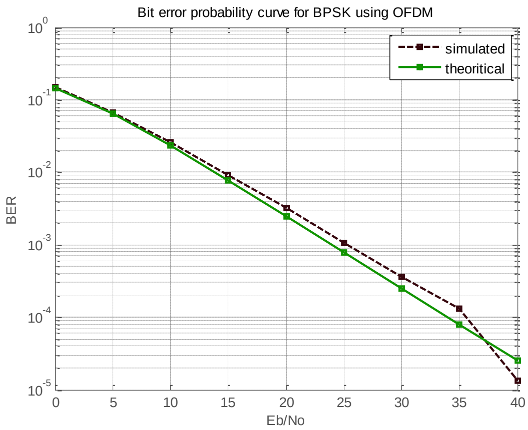

Figure 11: BER curve for OFDM-BPSK with Rayleigh Channel

IV. CONCLUSION

In this paper, the performance of BPSK with OFDM AWGN and Rayleigh fading distribution was evaluated. Graphical results show the improvement in BPSK with Rayleigh fading channel compared to its performance in AWGN channel. The graphical results prove that simulated BER of BPSK is same as that of theoretical BER of BPSK. The reported BER can be further reduced by using channel estimation or suitable diversity scheme.

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