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Continuous Time Markov Chains–An Optimization on Used Vehicle Sales

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Abstract- Using the basic properties involving the sojourn time and the transition rate between two states and the first passage time for a Continuous Time Markov Chain (CMTC) the applicability for optimizing the second sales used two wheelers in a showroom. Only the basic properties of the CTMC and the properties of rate matrix we are able to obtain a solution for maximum profit.

Keywords: Sojourn time, rate matrix, generator matrix, first passage time.

I. INTRODUCTION

Due to the fast moving nature of new type of vehicles, used vehicles do come to the market for sale which can be seen in all the regions of Tamil Nadu.
In view of this, this study is initiated in the Kanyakumari District using the CTMC. Here we have illustrated the working of the profit only with Markov chain properties.

II. METHODOLOGY

A stochastic process $\{X\{t\}, t \geq 0\}$ on the state space S is called a Continuous Time Markov Chain (CTMC) if, for all i and j in S and t, $s \geq 0$,

$$P\{X(s+t) = j \mid X(s) = i, X(0), 0 \leq u \leq s\} = P\{X(s+t) = j \mid X(s) = i\}$$

In this case the transition probability is,

$$P_{i,j} = P(X(t) = j \mid X(0) = i), 1 \leq i, j \leq N$$

And the t.p.m with N^2 entries is, (1.1)

$$P(t) = \begin{bmatrix} p_{1,1}(t) & p_{1,2}(t) & \cdots\cdots & p_{1,N}(t) \\ p_{2,1}(t) & p_{2,2}(t) & \cdots\cdots & p_{2,N}(t) \\ . & . & \cdots\cdots & . \\ p_{N,1}(t) & p_{N,2}(t) & \cdots\cdots & p_{N,N}(t) \end{bmatrix} \quad (1.2)$$

Suppose the state space of the stochastic process $\{ x(t), t \geq 0 \}$ is $\{ 1,2,3,\ldots,N \}$. Suppose the system starts in state i. It stays there for an Exp(r_i) amount of time , called the sojourn time in state i. At the end of the sojourn time in state i, the system makes a sudden transition to state $j \neq i$ with probability $P_{i,j}$, independent of how long the system has been in state i. Here Pi,i = 0, Since the sojourn time is the time till it moves out of state i. If i is absorbing, we set r_i=0, In such a case $P_{i,j}$ is have no meaning. Now a CTMC can be described by giving the parameters $\{ r_i, 1 \leq i \leq N \}$ and $\{P_{i,j}, 1 \leq i, j \leq N \}$.

The quantity r_i $P_{i,j}$ = $r_{i,j}$ is called the transition rate from i to is called the rate matrix of the CTMC. This matrix is closely related to the generator matrix ,

$$r_i = \sum_{j=1}^{N} r_{i,j} \quad (1.3)$$

j

$$P_{i,j} = \frac{r_{i,j}}{r_i} \quad if\ r_i \neq 0 \quad (1.4)$$

The matrix

$$R = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots\cdots & r_{1,N} \\ r_{2,1} & r_{2,2} & \cdots\cdots & r_{2,N} \\ . & . & \cdots\cdots & . \\ r_{N,1} & r_{N,2} & \cdots\cdots & r_{N,N} \end{bmatrix} \quad (1.5)$$

Q = [$q_{i,j}$] , where

$$q_{i,j} = \begin{cases} -r_i \ , if\ i = j \\ r_{i,j} \ , if\ i \neq j \end{cases} \quad (1.6)$$

Thus the generator matrix Q is the same as the rate matrix R with the diagonal elements replaced by $-r_i$ ‘s.

III. FIRST PASSAGE TIME

Let $\{X(t), t \geq 0 \}$ be a CTMC with state space $\{1,2,\ldots\ldots,N\}$ and rate matrix R. The first passage into state N is defined to the,

T=min $\{ t \geq 0 ; X(t) = N \}$ (1.8)

$$m_i = E(T \mid X(0) = i) \quad (1.7)$$

$[m_i, 1 \leq i \leq N-1]$ satisfy

$$r_i m_i = 1 + \sum_{j=1}^{N-1} r_{i,j} m_j, 1 \leq i \leq N-1 \quad (1.9)$$

Or

$$m_i = \frac{1}{r_i} + \sum_{j=1}^{N-1} \frac{r_{i,j} m_j}{r_i}$$

IV. APPLICATION

(a) Sampling

Due to the enormous out coming of new models of Two wheelers, the second

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sales showrooms have a mushroom growth in all the urban areas in almost all the towns. Kanyakumari District is very exceptional in the since that every road is flooded with this type of vehicles on the roads and hence this study is considered as a good fit.

The customers fix some price for their vehicle and leave it to the settlers in their showroom and the commission is at least 15 percent. There are 5 fast moving models which we designate as A, B, C, D, E one such seller is selected. His parking lot can hold atmost 100 two wheelers at a time.

(b) Results

The details of the price for the owner agent, weekly arrivals, weekly demand one presented below in Table 1.

Table 1: Data for used two wheeler bikes

Model	Owner's Price (Rs.)	Supply Bikes / Months	Selling Price (R.s)	Demand Bikes / Month	Profit
1	6,000	40	6,900	16	900
2	12,700	32	13,605	10	1905
3	16,400	14	18,860	8	2460
4	19,500	13	22,425	12	2925
5	21,500	7	24,725	5	3225

we shall now maximize the net profit of the seller. The cost of the inventory carrying charge includes the rent of the building, display charge, his salary and employees salaries which is assumed to be the 2% of the average inventory per month.

With this we shall construct the objective function. Let p_j be the limiting distribution of the $\{ X(t), t \geq 0 \}$ process. It is given by,

$$P_i = \frac{1-\rho}{1-\rho^{k+1}} \rho^i , 0 \leq i \leq k \qquad (1.10)$$

Where $\rho = \frac{\lambda}{\mu}$, λ is the arrivalintensity, μ is the departureintensity

Thus the expected number of model 1 on this steady state is,

$$L = \sum_{j=1}^{K_1} jP_j$$

Since each bike costs Rs 6,000, the average value of the inventory is Rs.6000L. Hence the inventory carrying cost per month is 0.02 x 6000 x L = 120L.

The dealer sells bikes at the rate of 16 in state 1 through K_1, the stock capacity for model 1 and no vehicles are sold in state 0. Hence the monthly rate of sales is $16(1-P_0)$. The dealer purchases model 1 at the rate of 40 in states 0 through K_1-1, and no bike are purchased at K_1. Hence the monthly purchase from type I is $40(1-P_{K_1})$. Clearly the two rates must be identical. Each bike generates a profit of .15 x 6000 = 900 Rs. Hence the profit rate in steady state is $900 \times 16 \times (1-P_0) = 900 \times 40 \times (1-P_{K_1})$. Hence the net profit rate in Rs per month is, $g_1(K_1) = 14400\,(1-Po) - 120L$

Both L and P_0 are functions of K_1. Hence g_1 in a function of K_1. Drawing the graph it is concave w.r.to K_1 and the maximum is at K_1=13, This implies that the optimal stock is 13 in model 1 bike. In a similar way the optimal stock level for the other models of vehicles are,

model 2 = 11
model 3 = 7
model 4 = 10
model 5 = 5

But the actual stock depends on the profit function for other models.

Now, the monthly profit from the following static stocking policy $K = [K_1, K_2, \ldots, K_5]$ is given by,

$$g(K) = \sum_{i=1}^{5} g_i(K_i)$$

Thus, the optimal static stocking policy can be obtained by solving maximization problem.

max g(K)
subject to

$$\sum_{i=1}^{5} K_i \leq 100$$

and $K_i \geq 0$ are integers.

We solve this problem by the simple greedy algorithm. The algorithm is,

Step 1:- Initialize K=[0,0,0,0,0]

Step 2:-Compute $\Delta_i = g_i(K_i+1) - g_i(K_i)$, $1 \leq i \leq 5$

Let r=argmax $\{ \Delta_i , 1 \leq i \leq 5 \}$

Step 3:-If ,

$$\sum_{i=1}^{5} K_i < 100 \text{ and } \Delta_r > 0 \text{ suchthat}$$

$$K_r = K_r + 1$$

and go back to step 2 .

Step4:-

$$\text{if } \sum_{i=1}^{5} K_i = 100 \text{ or } \Delta_r \leq 0, \text{ stop}$$

K is the optimal allocation vector

Table 2: Results for the used Bike sale dealer

Policy	Allocation Vector	Expected Monthly Profit
Son	[18,26,28,28,10]	2 1 9 5 1 0
Father	[20,21,22,19,18]	2 2 5 7 5 0
Optimal	[16,17,22,23,22]	2 3 9 1 3 0

Thus the optimal allocation vector is [16,17,22,23,22]. The son's policy does not do well, Father's a little close to optimum and the optimal is the best.

V. CONCLUSIONS

The study reveals that if the annual rates and demand for sales and the % commission are the same as what is got from the used bike sellers in the city, This business is a profitable enterprise and does not require any high level of mathematics.

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