International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249 – 8958, Volume-2, Issue-4, April 2013

Wavelet Collocation Solution for Convective Radiative Continuously Moving Fin with Temperature-Dependent Thermal Conductivity

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Abstract: ***In this paper, the temperature distribution in convective radiative continuously moving fin with variable thermal conductivity, which is loosing heat by simultaneous convection radiation to surroundings, is studied. We consider three particular cases, namely thermal conductivity is (I) constant (II) a linear function of temperature and (III) an exponential function of temperature. Wavelet Collocation Method is used to solve this nonlinear heat transfer problem. The exact solution obtained in absence of radiation- conduction fin parameter are compared with Wavelet Collocation solution are same. The fin efficiency is computed in absence of radiation-conduction fin parameter. The whole analysis is presented in dimensionless form and effect of different parameters such as thermal conductivity parameter 'a', Peclet number 'Pe', convection-conduction parameter 'N_c', radiation-conduction parameter 'N_r', dimensionless convection sink temperature 'θ_a' and dimensionless radiation sink temperature 'θ_s' on the fin temperature is discussed in detail.***

Keywords: Wavelet, Collocation, Convection, Radiation, heat transfer, fin, Conductivity.

I. INTRODUCTION

Extended surfaces are widely used in various industrial applications. In processes such as optical fiber, hot rolling and casting, the material being manufactured is processed thermally by allowing it to exchange heat with the ambient while it is in continuous motion.

A brief review of such type of problems are studied by several authors. Kraus et.al [1] presented a comprehensive review on this topic. To reduce the mathematical complexity he assumed constant thermophysical properties and uniform heat transfer coefficient and obtained closed form analytical solutions for a number of cases. Karwe and Jaluriya [2] used numerical method to solve coupling problem of heat transfer in the moving material and the transport in the fluid. The thermal fields in the material and in the fluid are computed. They observed that the temperature level decay gradually with distance along the moving material. Y. Jaluria, A.P. Singh [3] solved the same problem numerically. Effect of Biot number and Peclet number on temperature distribution in the material and the surface heat loss studied.

Manuscript received April. 2013.
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S. R. Choudhury, y. Jaluria [4] found an analytical solution in the form of an infinite series for the transient temperature distribution in a flat plate and in a cylindrical rod of finite length moving at a constant speed. First 25 terms of the series was found to be sufficient to obtain converged solution.

They compared analytical solutions with numerical solutions and found excellent agreement between these two. Mohsen Torabi et. al [5] studied heat transfer in a moving fin with variable thermal conductivity, which releases heat by simultaneous convection and radiation to its surroundings. Differential transform Method used in solution and the effect of different parameters also studied. A. Aziz and Robert J. Lopez [6] studied the heat transfer process in a continuously moving sheet or rod of variable thermal conductivity that releases heat by simultaneous convection and radiation. The process is governed by five dimensionless parameters Pe, a, Nc, Nr, and θ_a. They used Runge- Kutta-Fehlberg method and effect of these five parameters studied in detail. Abdul Aziz and F. Khani [7] studied convection-radiation of a continuously moving fin of variable thermal conductivity and used homotopy analysis method. Effect of different parameters also studied.

No solution is available when thermal conductivity varies in general with temperature. In this study an attempt has been made to solve nonlinear boundary value problem describing the process of heat transfer through continuously moving fin with variable thermal conductivity using the Wavelet Collocation Method. We consider the heat transfer in a fin whose thermal conductivity varies in general with temperature and the heat transfer coefficient considered constant. Three particular cases when thermal conductivity is (I) constant (II) a linear and (III) an exponential function of temperature are studied in detailed. In this study we consider one special case when $N_r = 0$ and found exact solution. We compare Exact and WCM results and observe that both results are same.

Nomenclature

A	fin cross section area (m^2)
C_p	specific heat of the material ($J.kg^{-1}.k^{-1}$)
h	convection heat transfer coefficient ($wm^{-2}k^{-1}$)
$K(T)$	temperature-dependent thermal conductivity ($wm^{-1}k^{-1}$)
K_a	thermal conductivity at the temperature T_a ($wm^{-1}k^{-1}$)
L	fin length (m)
P	fin perimeter (m)
T	Local fin temperature(k)
T_a	sink temperature for convection (k)
T_b	fin base temperature (k)
T_s	sink temperature for radiation (k)
U	speed of moving fin ($m.s^{-1}$)
x	axial distance measured from the tip of fin (m)
Dimensionless parameters	
a	dimensionless thermal conductivity parameter
exp	exponential function
N	fin parameter

N_c	convection-conduction parameter
N_r	radiation-Conduction parameter
Pe	Peclet number
X	axial distance measured from the tip of fin
Greek symbols	
α	thermal diffusivity of the material ($m^2.s^{-1}$)
β	slope of the thermal conductivity-temperature curve (k^{-1})
ε	emissivity
θ	dimensionless temperature
θ_a	dimensionless convection sink temperature
θ_b	dimensionless fin's base temperature
θ_s	dimensionless radiation sink temperature
ρ	density of material (kg. m^{-3})
σ	Stefan-Boltzmann constant
η	fin efficiency
Abbreviation	
ADM =	Adomian Decomposition Method
HAM =	Homotopy Analysis Method
HPM =	Homotopy Perturbation Method
WCM =	Wavelet Collocation Method

II. FORMULATION OF THE PROBLEM

In Fig.1 geometry of moving fin is given. We consider the thermal processing of a plate or a rod of cross-sectional area A and perimeter P while it moves horizontally with a constant speed U. The hot plate or rod emerges from a die or furnace at a constant temperature T_b. The motion of the plate or rod may induce a flow field in an otherwise quiescent surrounding medium or alternatively, the plate or rod may experience and externally driven flow over its surface. We consider hot plate or rod release heat in surrounding medium by convection and radiation. The radiative component would play a more prominent role if the forced convection is weak or absent or when only natural convection occurs. The convection and radiation sink temperatures are taken to be different, and one can be varied independent of the other. We assume sink temperature $T_a < T_b$ for both convection and radiation. The surface of moving material is assumed to be gray with a constant emissivity ε. The convective flow in the surrounding medium provides a constant heat transfer coefficient h over the entire surface of the moving material. As the material undergoing the treatment experiences a large change in its temperature during the process, the change in its thermal conductivity may be as much as 100%. For most materials, the thermal conductivity of the material k increases linearly with temperature. In this study we consider thermal conductivity is the general function of temperature. Three particular cases studied for thermal conductivity (I) constant (II) linear and (III) an exponential function of temperature.

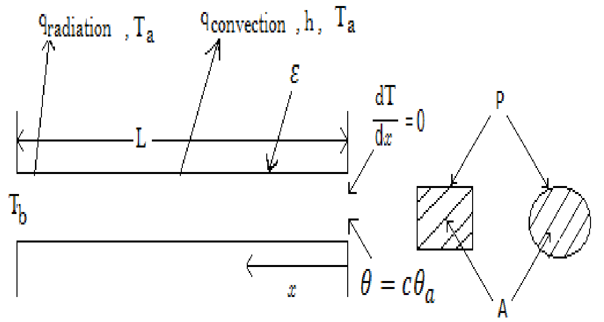

Fig.1 Geometry of moving fin

The steady-state energy balance for the material moving with a constant speed and releasing heat by simultaneous convection and radiation may be written as

$$\frac{d}{dx}\left(K(T)\frac{dT}{dx}\right)-\frac{hP}{A}(T-T_a)-\frac{\varepsilon\sigma P}{A}(T^4-T_s^4)+\frac{1}{\alpha}U\frac{dT}{dx}=0 \quad (1)$$

where α $=K_a/(\rho C_p)$ is the thermal diffusivity of the material, ρ is the density, and C_p is the specific heat. The last term on the left in Equation (1) is the advection term. The axial coordinate x is measured from the tip of the moving fin. Boundary conditions are

$$x=0, \quad \frac{dT}{dx}=0 \quad (2)$$

$$x=L, \quad T=T_b \quad (3)$$

we consider thermal conductivity is a general function of temperature

$$K(T) = K_a f(\theta) \quad (4)$$

Introducing dimensionless variables and similarity criteria.

$$\theta=\frac{T}{T_b}, \quad \theta_a=\frac{T_a}{T_b}, \quad \theta_s=\frac{T_s}{T_b}, \quad L^*=\frac{PL}{A}, \quad x=\frac{xL^*}{L},$$

$$N_c=\frac{hA}{PK_a}, \quad a=\beta T_b, \quad N_r=\frac{\varepsilon\sigma A T_b^3}{PK_a}, \quad Pe=\frac{UA}{\alpha P} \quad (5)$$

Assuming $L^* = 1$, the system of equations (1) to (3) reduces to the following form

$$f(\theta)\frac{d^2\theta}{dX^2}+f'(\theta)\left(\frac{d\theta}{dX}\right)^2-N_c(\theta-\theta_a)-N_r(\theta^4-\theta_s^4)+Pe\frac{d\theta}{dX}=0, 0\le x\le 1 \quad (6)$$

$$x=0, \frac{d\theta}{dX}=0 \quad (7)$$

$$x=1, \quad \theta=1 \quad (8)$$

III. SOLUTION OF PROBLEM

We have solved this problem using Wavelet Collocation Method. Here we have used nine Legendre wavelet basis functions in solution. MATLAB software has been used in solution.

Wavelets:Continuous wavelets are defined by the following formula:

$$\psi_{a,b}(x) = |a|^{-\frac{1}{2}}\psi\left(\frac{x-b}{a}\right), \quad a,b\in R, \; a\neq 0 \quad (9)$$

where a is dilation parameter and b is a translation parameter. Legendre wavelets $\psi_{n,m}(x) = \psi(k,\hat{n},m,x)$, where $\hat{n}$ =2n-1, n = 1, 2,… ,2^{k-1}, k is any positive integer, m is the order of Legendre polynomials and x is the normalized time.

Legendre wavelets defined on the interval (0, 1) by

$$\psi_{n,m}(x)=\begin{cases}\sqrt{(m+1/2)}\,2^{\frac{k}{2}}P_m(2^kx-\hat{n}), & \frac{\hat{n}-1}{2^k}\le x<\frac{\hat{n}+1}{2^k}\\ 0 & \text{otherwise}\end{cases} \quad (10)$$

where $m = 0, 1,\ldots,M-1$ and $n = 1, 2,\ldots,2^{k-1}$. Here $P_m(x)$ is the well known Legendre polynomials of order m.

$$P_0(x) = 1,\ P_1(x) = x,\ P_{m+1}(x) = \frac{2m+1}{m+1} x\, P_m(x) - \frac{m}{m+1} P_{m-1}(x)$$
m =1, 2, 3,…, M-1 (11)

A function f(x) defined in domain [0, 1] can be expressed as

$$f(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\, \psi_{n,m}(x) \quad (12)$$

Where $c_{n,m} = \langle f(x), \psi_{n,m}(x) \rangle$ in which < .,. > denotes the inner product.

If we take some terms of infinite series, then (12) can be written as

$$f(x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\, \psi_{n,m}(x) = C^T\psi(x) \quad (13)$$

Where C and ψ(x) are M× 1 Matrices given by

$$C = [c_{10}, c_{11}, \ldots, c_{1M-1}, c_{20}, c_{21} \ldots, c_{2M-1}, \quad c_{2^{k-1}0}\, c_{2^{k-1}1}, \ldots, c_{2^{k-1}M-1}\ ]^T \quad (14)$$

$$\psi(x) = [\psi_{10}(x), \psi_{11}(x), \ldots, \psi_{1M-1}(x), \psi_{20}(x), \ldots, \psi_{2M-1}(x), \ldots, \psi_{2^{k-1}0}(x)\, \psi_{2^{k-1}1}(x), \ldots, \psi_{2^{k-1}M-1}(x)]^T \quad (15)$$

(i) Property of the product of two Legendre wavelets

If E is a given wavelets vector then we have the property

$$E^T\psi\psi^T = \psi^T\hat{E} \quad (16)$$

where $\hat{E}$ is M × M matrices depend on the wavelet vector E

(ii) Operational matrix of integration: The integration of the wavelets $\psi(x)$ which is defined in (10) can be obtained as [8].

$$\int_0^x \psi(s)ds = P\,\psi(x), x \in [0, 1] \quad (17)$$

where P is $2^{k-1}M\times 2^{k-1}M$, k=1, operational matrix of integration given by

$$P = \frac{1}{2}\begin{bmatrix} 1 & \frac{1}{\sqrt{3}} & 0 & 0 & & 0 & 0 \\ \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{15}} & 0 & \cdots & 0 & 0 \\ 0 & \frac{-1}{\sqrt{15}} & 0 & \frac{1}{\sqrt{35}} & & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{35}} & 0 & & 0 & 0 \\ & \vdots & & & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & \cdots & 0 & \frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \\ 0 & 0 & 0 & 0 & & \frac{-\sqrt{2M-1}}{(2M-1)\sqrt{2M-3}} & 0 \end{bmatrix} \quad (18)$$

3.1 Wavelet Collocation Method

Let
$$\theta'' = C^T\psi(x) \quad (19)$$

Integrating equation (19) with respect to x from 0 to x, we have $\theta'(X) = \theta'(0) + C^TP\psi(x)$

$\theta'(X) = C^TP\psi(x)$ since $\theta'(0) = 0$ (20) Again Integrating equation (20) with respect to x from 0 to x, we have

$$\theta(X) = \theta(0) + C^TP^2\psi(X) \quad (21)$$

At point X=1 we get

$$\theta(1) = \theta(0) + C^TP^2\psi(1)$$
$\theta(0) = 1 - C^TP^2\psi(1)$ using in equation (21) we have

$$\theta(X) = 1 - C^TP^2\psi(1) + C^TP^2\psi(X) \quad (22)$$

From equation (6) to (8) we get

$$f(\theta)C^T\psi(X) + f'(\theta)(C^TP\psi(X))^2 - N_c(1 - C^TP^2\psi(1) + C^TP^2\psi(X) - \theta_a) - N_r\{1 - C^TP^2\psi(1) + C^TP^2\psi(X)\}^4 - \theta_s^4 + Pe(C^TP\psi(x)) = 0 \quad (23)$$

We study the equation (23) in three particular cases which are given below.

Case I: In this case we consider $f(\theta) = 1$ then equation (23) reduce to

$$C^T\psi(X) - N_c(1 - C^TP^2\psi(1) + C^TP^2\psi(X) - \theta_a) - N_r[\{1 - C^TP^2\psi(1) + C^TP^2\psi(X)\}^4 - \theta_s^4] + Pe\big(C^TP\psi(x)\big) = 0 \quad (24)$$

Case II: In this case we consider $f(\theta) = 1 + a(\theta - \theta_a)$ then equation (23) reduce to

$$\{1 + a\,(1 - C^TP^2\psi(1) + C^TP^2\psi(X) - \theta_a)\}\ C^T\psi(X) + a(C^TP\psi(X))^2 - N_c(1 - C^TP^2\psi(1) + C^TP^2\psi(X) - \theta_a) - N_r[\{1 - C^TP^2\psi(1) + C^TP^2\psi(X)\}^4 - \theta_s^4] + Pe(C^TP\psi(x)) = 0 \quad (25)$$

Case III: In this case we consider $f(\theta) = \exp(a(\theta - \theta_a))$ then equation (23) reduce to

$$\exp(a(1 - C^TP^2\psi(1) + C^TP^2\psi(X) - \theta_a))\, C^T\psi(X) + a\exp(a(1 - C^TP^2\psi(1) + C^TP^2\psi(X) - \theta_a))\,(C^TP\psi(X))^2 - N_c(1 - C^TP^2\psi(1) + C^TP^2\psi(X) - \theta_a) - N_r[\{1 - C^TP^2\psi(1) + C^TP^2\psi(X)\}^4 - \theta_s^4] + Pe(C^TP\psi(x)) = 0 \quad (26)$$

.

As θ (x) is an approximate solution of system (6), (7) and (8). Choosing collocation points x_i , $i = 1,2,3, \ldots, n$ in the interval [0, 1].

Solving system of equations separately (24), (25) and (26) for case I, Case II and Case III respectively, using present Wavelet Collocation Method with the help of MATLAB software.

Special case I: Exact solution when thermal conductivity is constant and $N_r = 0$.

Putting f(θ) = 1 and $f'(\theta) = 0$ in equation (6)

$$\frac{d^2\theta}{dX^2} + Pe\frac{d\theta}{dX} - N_c(\theta - \theta_a) = 0,\ 0 \le x \le 1$$

$$\theta(x) = c1e^{m1x} + c2e^{m2x} + \theta_a$$

where $m_1 = \frac{-Pe+\sqrt{Pe^2+4N_c}}{2}$, $m_2 = \frac{-Pe-\sqrt{Pe^2+4N_c}}{2}$,

$c1 = \frac{-m_2(1-\theta_a)}{m_1e^{m_2}-m_2e^{m_1}}$ and $c2 = \frac{m_1(1-\theta_a)}{m_1e^{m_2}-m_2e^{m_1}}$

IV. FIN EFFICIENCY

Fin efficiency is defined by $\eta = \frac{KA}{PhL}\left(\frac{dT}{dx}\right)_{at\ x=L}$ using dimensionless variables, we get following formulae**.** In special case I fin efficiency is defined by

$$\eta = \frac{1}{N_c}\theta'(1),$$

V. RESULT AND DISCUSSION

For checking the accuracy of WCM solution, we obtain exact solution in special 'case I' when thermal conductivity is constant and $N_r = 0$. In this case we compare exact and WCM results, and observed that both results are same [Table1]. In fig.2 to 14 only one parameter is varied and keeping the other parameters fixed. The following values were used in computation $a = 0.5$, $N_c = 0.25$, $N_r = 0.75$, $Pe = 0.5$, $\theta_a = 0.4$, $\theta_s = 0.4$.

Table 1 Comparison of exact and WCM results for Nr = 0

Value of parameters	Exact	WCM
Pe = 0, Nc =0.25	0.8868188840	0.8868188840
Pe = 0.25, Nc =0.25	0.8947889165	0.8947889165
Pe = 0.50, Nc =0.25	0.9019645600	0.9019645600
Pe = 0.25, Nc =0. 50	0.8064510261	0.8064510261

and constant thermal conductivity

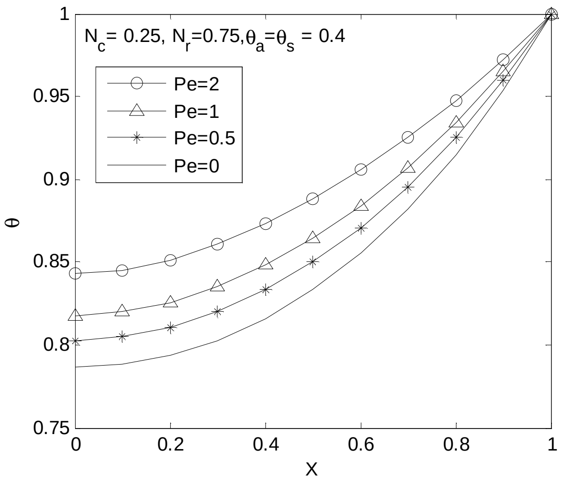

Fig. 2 Case I Effect of Pe on temperature distribution in moving fin

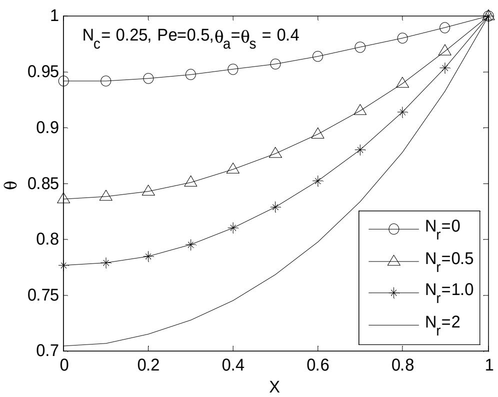

Fig. 3 Case I Effect of N_r on temperature distribution in moving fin

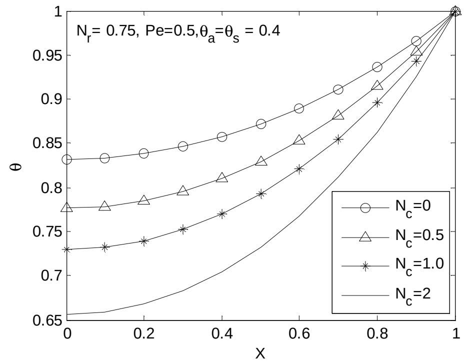

Fig. 4 Case I Effect of N_c on temperature distribution in moving fin

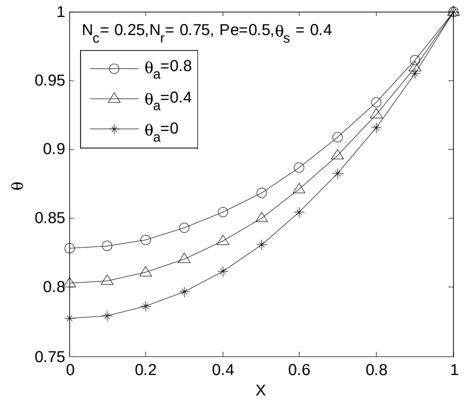

Fig. 5 Case I Effect of θ_a on temperature distribution in moving fin

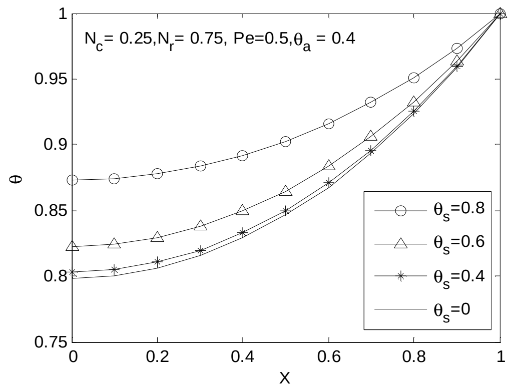

Fig. 6 Case I Effect of θ_s on temperature distribution in moving fin

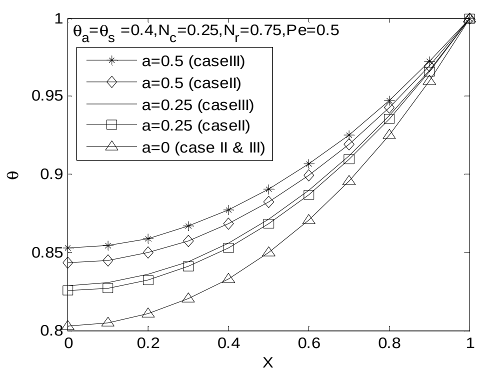

Fig.7 Case II & III Effect of thermal conductivity on temperature distribution in moving fin

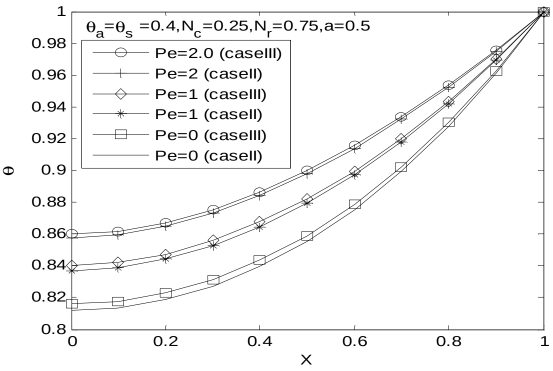

Fig.8. Case II & III Effect of Peclet number on the temperature distribution in moving fin

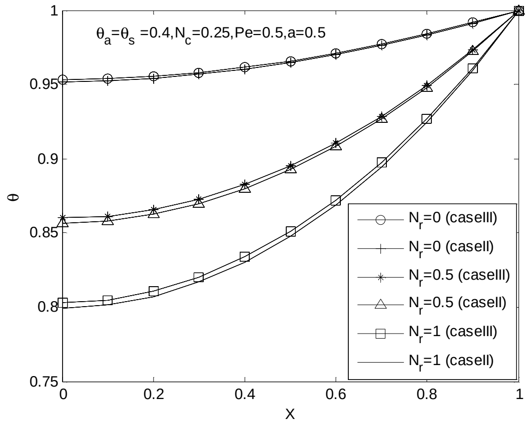

Fig.9. Case II & III Effect of radiation-conduction parameter on the temperature distribution in moving fin

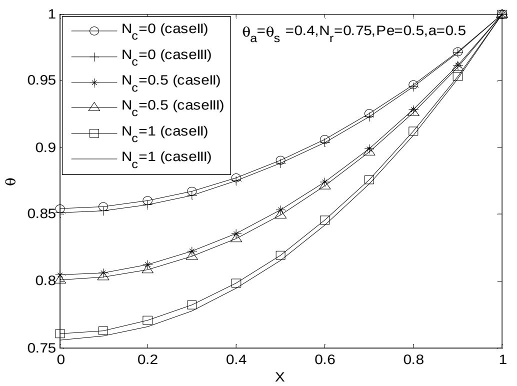

Fig.10. Case II & III Effect of convection-conduction parameter on the temperature distribution in moving fin

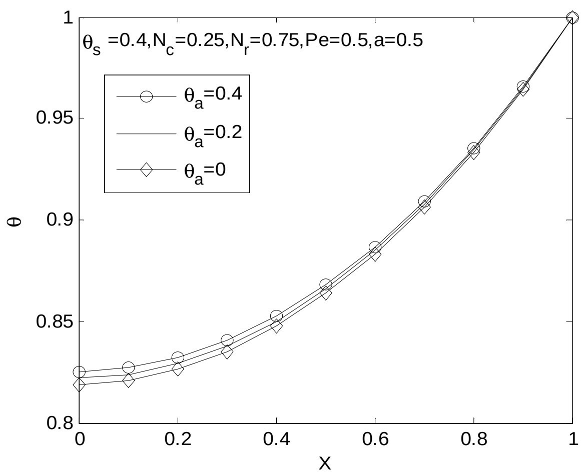

Fig.11. Case II Effect of convection sink temperature on the temperature distribution in moving fin

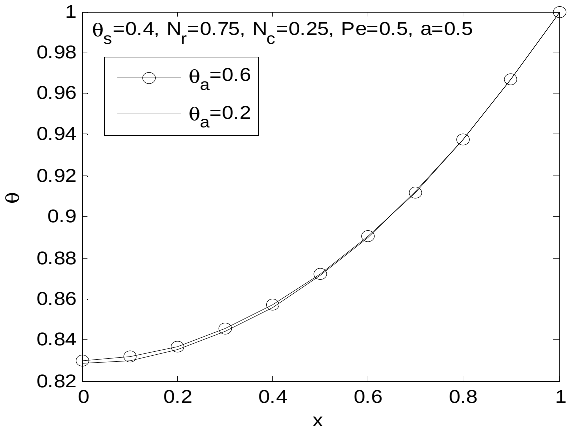

Fig.12. Case III Effect of convection sink temperature on the temperature distribution in moving fin

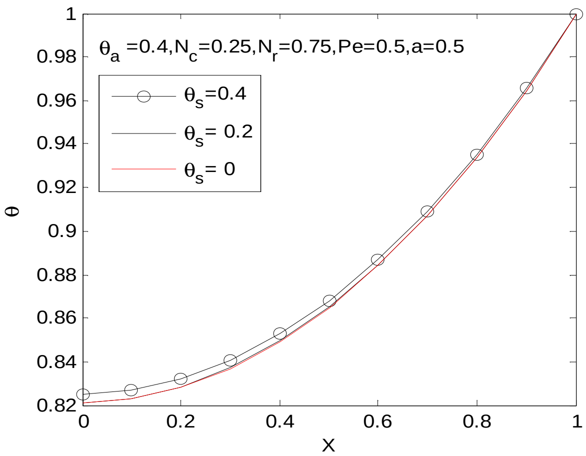

Fig.13. Case II Effect of radiation sink temperature on the temperature distribution in moving fin

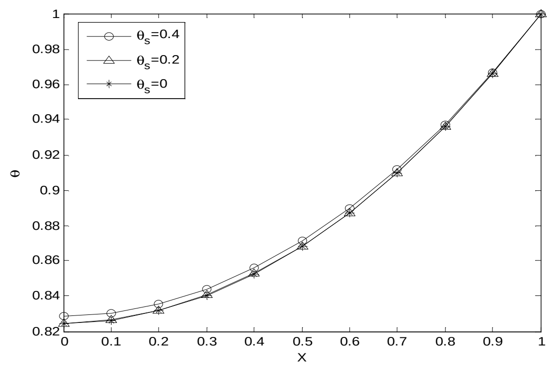

Fig.14. Case III Effect of radiation sink temperature on the temperature distribution in moving fin

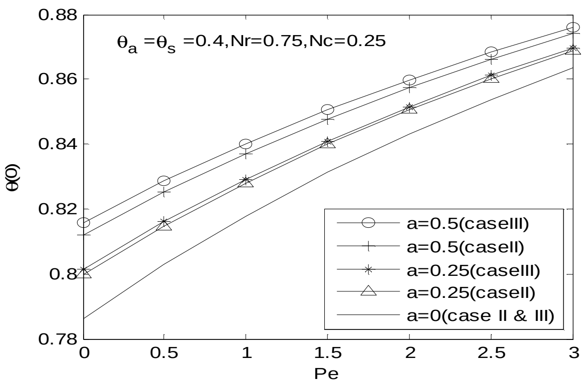

Fig.15. Case II & III Effect of Peclet number on the fin-tip temperature in moving fin

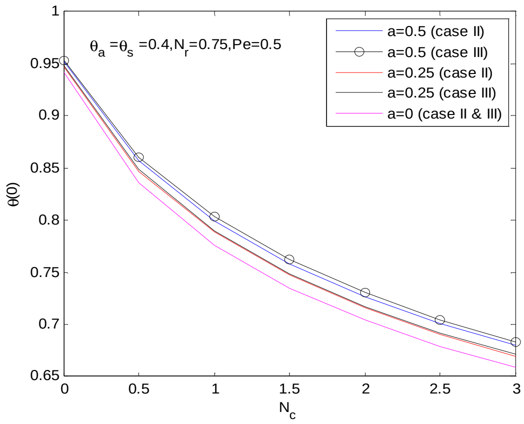

Fig.16. Case II & III Effect of convection-conduction parameter on the fin-tip temperature in moving fin

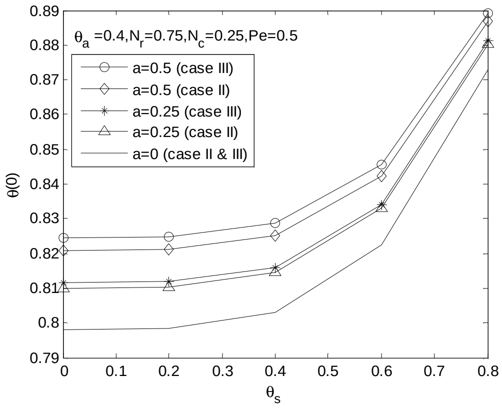

Fig.17. Case II & III effect of radiation sinks temperature on the fin-tip temperature in moving

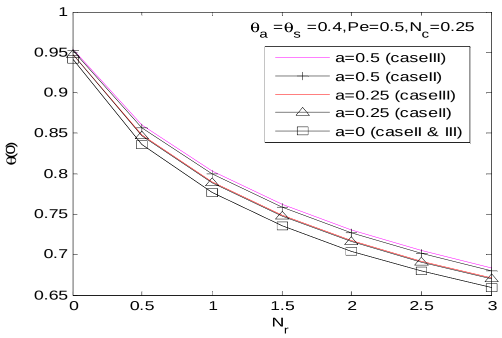

Fig.18. Case II & III Effect of radiation-conduction parameter on the fin-tip temperature in moving fin

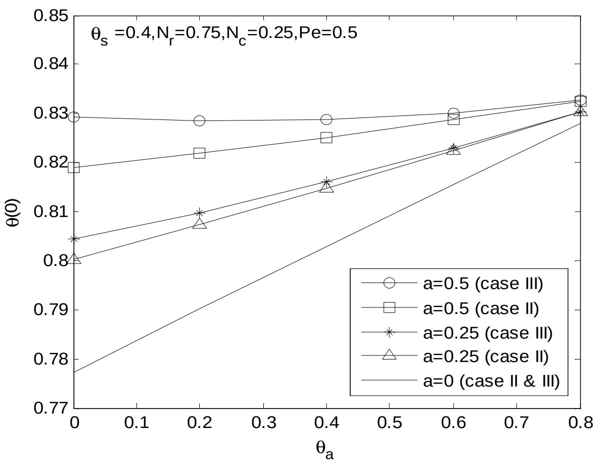

Fig.19. Case II and III Effect of sink temperature on the fin-tip temperature in moving fin

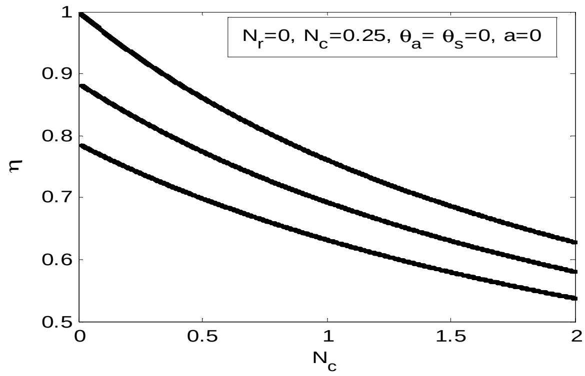

Fig.20 case 1 Effect of Pe on fin efficiency for $N_r = 0$, Pe = 0, 0.25, 0.5 top to bottom respectively

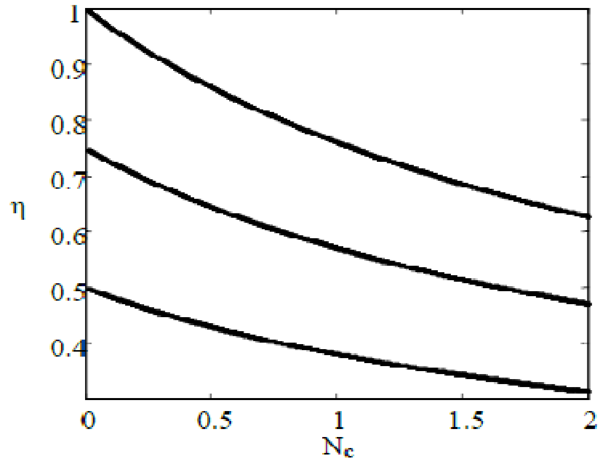

Fig.21. Special case 1 Fin efficiency for Pe=0, $N_r = 0$, $\theta_a = 0$, 0.25, 0.5 top to bottom respectively

Peclet number is the ratio of heat transport by convection and heat transport by conduction. As Pe increases heat transport by convection is increases and temperature of fin increases. Further as Pe decreases heat transport by conduction is increases and temperature of fin decreases. Thus as peclet number increases, convection becomes more dominant in comparison to conduction. And because of it fin efficiency decreases as shown in figure 20. Figure 2 represents the effect of the peclet number on the temperature distribution in the fin. For constant thermal conductivity as Pe increases, i.e. fin moves faster, the time for which the material is exposed to the environment gets shorter, consequently temperature in fin is higher. And speed of fin is slow, the time for which the material is exposed to the environment gets longer, consequently temperature in moving fin is lower. The effect of the convection-conduction parameter N_c on temperature distribution in fin is shown in figure 3. As N_c increases, i.e. as convection gets stronger, the cooling becomes more effective, consequently temperature in fin decreases. Figure 4 represents the effect of the radiation-conduction parameter N_r on the temperature distribution in the fin. As heat radiation becomes higher, the radiative cooling becomes more effective, consequently temperature in moving fin is decreases. The effect of convection sink temperature θ_a on temperature distribution in fin is shown in Figure 5. As θ_a increases, convective heat loss from the moving fin to the environment is decreases consequently temperature in fin increases. Fig. 6 shows that the effect of radiation sink temperature θ_s on temperature distribution in moving fin. As θ_s increases, the radiative heat loss decreases consequently temperature in fin increases.

In cases II and III the effect of different parameters on the temperature in fin is presented by figures 7 to 19. Fig. 7 represents the effect of thermal conductivity on temperature distribution in fin. Thermal conductivity at base temperature is higher than the thermal conductivity at the environment temperature. As value of 'a' increases the average temperature in fin increases.

In Fig.8 we observed that as we increase value of Packet number, temperature in fin increases due to high speed of moving fin. In Fig.9 effect of radiation-conduction parameter N_r studied we observed that as we increase value of N_r temperature in fin decreases due to higher radiation. In Fig. 10 as we increase value of N_c we found that the temperature distribution in fin decreases due to high convection, the cooling becomes more effective in exponential case. Effect of convection sink temperature 'θ_a' on temperature distribution in fin is presented by figures 11 and 12 in case II and III respectively. We observe that the convection sink temperature θ_a increases, the convective heat loss from the moving fin decreases. Thus temperature distribution in fin increases form case II to III. Figures 13 and 14 represents the effect of the radiation sink temperature θ_s on temperature in the moving fin, as we increase θ_s temperature in fin increases due to radiative heat loss. Temperature in moving fin increases from case II to III.

In figure 15 fin tip temperature increases as the peclet number and the thermal conductivity parameter increases, explanation is similar as given for figures 2 and 7. In this figure we observe that fin tip temperature in case III is higher than case II. Effect of N_c on fin tip temperature is presented in figure16. We observed that as value of N_c increases, fin tip temperature decreases. As we increase value of 'a' with N_c temperature also increases form case II to III. In figure 17 tip temperature increases as radiation-sink temperature and thermal conductivity of fin material increases.

Effect of radiation-conduction and thermal conductivity parameter on fin tip temperature is presented in figure 18. In this figure tip temperature decreases as value of N_r increases, for higher thermal conductivity temperature is higher and increases from case II to III. In figure 19 we observe that if we increase sink temperature and thermal conductivity of moving fin, the tip temperature of fin increases. Detailed explanation of increasing or decreasing temperature in fin is given in case I.

The fin efficiency for $N_r = 0, N_c = 0.25,\ a = 0,\ \theta_a = \theta_s = 0$ by WCM and exact method both are same and presented in figures 20 and 21. we observe that the fin efficiency decreases as value of N_c increases. Previously we explain that Lower values of Pe and θ_a gives lower temperature in fin; consequently fin efficiency is high for lower value of Pe and θ_a it confirms from figure 20 and 21.

VI. CONCLUSION

When thermal conductivity is constant and N_r = 0, we compare exact and WCM results and observe that both results are same [table 1]. In special case I, efficiency of fin is same for WCM and exact method. This confirms the superiority of the method, related to the accuracy level. Thus WCM gives high accuracy and non linear problems can be easily solved.

Effect of different parameters studied in detail, as we increase value of Pe, a, θ_a, θ_s temperature in moving fin increases. As fin material moves faster, cooling process becomes slower. As convection-conduction and radiation-conduction increases, cooling process is faster. Average thermal conductivity of fin increases due to high thermal conductivity at base, therefore temperature in fin increases as 'a' increases. Fin efficiency is high for lower value of Pe and θ_a, presented by figures 20 and 21 respectively. In special case I, efficiency of fin is same for WCM and exact method. Thus WCM gives high accuracy and nonlinear problem can be easily solved.

VII. ACKNOWLEDGEMENT

1. Authors are grateful to Professor Umesh Singh Co-ordinator DST- CIMS BHU Varanasi, India for providing necessary facilities.
2. Author Surjna Singh is grateful to Gurgaon Institute of Technology and Management Gurgaon (Haryana), India for sponsorship.

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