Binary Decision Diagram based Reliability Evaluation

Manoj Singhal

***Abstract*— In this paper, I have considered a computer communication network which has perfect vertices and imperfect links. It means communication links may fail with known probability. I have found the reliability of the given network by using an exact method (inclusion-exclusion formula) and with binary decision diagram. I have found that the reliability obtained by both the method is same. Binary decision diagram based reliability evaluation involves three main steps. First ordering the given communication link by applying a heuristic approach. I have proposed a heuristic approach to generate the minimum size binary decision diagram. Second generate the reliability function with the help of min-paths from source to sink. At last apply Shannon's decomposition to compute the reliability of the given network.**

***Index Terms*— Binary Decision Diagrams (BDD), Directed Acyclic Graph (DAG), Computer communication Network (CNN), Modified Binary Decision Diagram (MBDD), Ordered Binary Decision Diagram (OBDD), Dual Binary Decision Diagram (DBDD).**

I. INTRODUCTION

Network reliability analysis receives considerable attention for the design, validation, and maintenance of many real world systems, such as computer, communication, or power networks. The components of a network are subject to random failures, as more and more enterprises become dependent upon computer communication network (CCN) or networked computing applications. Failure of a single component may directly affect the functioning of a network. So the probability of each component of a CCN is a crucial consideration while considering the reliability of a network. Hence the reliability consideration is an important factor in CCN. The IEEE 90 standard defines the reliability as "the ability of a system or component to perform its required functions under stated conditions for a specified period of time." There are so many exact methods for computation of network reliability [1]. The network model is a directed stochastic graph G = (V, E), where V is the vertex set, and E is the set of directed edges. An incidence relation which associates with each edge of G a pair of nodes of G, called its end vertices. The edges represent components that can fail with known probability. In real problems, these probabilities are usually computed from statistical data.

The problem related with connection function is NP-hard. The same thing is observed for planar graphs [12]. In the exact method there are two classes for the computation of the network reliability. The first class deals with the enumeration of all the minimum paths or cuts. A path is a subset of components (edges and/or vertices), that guarantees the source and the sink to be connected if all the components of this subset are functioning. A path is a minimal if a subset of elements in the path does not exist that is also a path. A cut is a subset of components (edges and/or vertices), whose failure disconnect the source and sink. A cut is a minimal if the subset of elements in the cut does not exist that is also a cut [26, 27]. The probabilistic evaluation uses the inclusion-exclusion, or sum of disjoint products methods because this enumeration provides non-disjoint events. Numerous works about this kind of methods have been presented in literature [6].

In the second class, the algorithms are based on graph topology. In the first process we reduce the size of the graph by removing some structures. These structures as polygon-to-chain [13] and delta-to-star reductions [11]. By this we will be able to compute the reliability in linear time and the reduction will result in a single edge. The idea is to decompose the problem in to one failed and another functioning [10]. The same was confirmed by Theologou & Carlier [23] for dense networks. Satyanarayana & Chang [4] and Wood [25] have shown that the factoring algorithms with reductions are more efficient at solving this problem than the classical path or cut enumeration methods.

II. BINARY DECISION DIAGRAMS

Akers [5] first introduced binary decision diagrams (BDD) to represent Boolean functions i.e. a BDD is a data structure used to represent a Boolean Function. Bryant [24] popularized the use of BDD by introducing a set of algorithms for efficient construction and manipulation of BDD structure. The BDD structure provides compact representations of Boolean expressions. A BDD is a directed acyclic graph (DAG) based on the Shannon decomposition. The Shannon decomposition for a Boolean function is defined as follows:

$$f = x.\, f_{x=1} + \bar{x}.\, f_{x=0}$$

Where x is one of the decision variables, and f is the Boolean function evaluated at x = i. By using Shannon's decomposition, any Boolean expression can be transformed in to binary tree. BDD are used to work out the terminal reliability of the links. Madre and coudert [22] found BDD usefulness in reliability analysis which was further extended by Rauzy [2, 3]. They are specially used to assess fault trees in system analysis.

In the network reliability framework, Sekine & Imai [9] have shown how to functionally construct the corresponding BDD. An alternate approach was shown by Singhal, Chauhan and Sharma [21] to compute BDD based network reliability.

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Table 1 shows the truth table of a Boolean function f and its corresponding Shannon tree is shown in figure 1. Sink nodes are labelled either with 0, or with 1, representing the two corresponding constant expressions. Each internal node u is labelled with a Boolean variable var(u), and has two out-edges called 0-edge, and 1-edge.

The node linked by the 1-edge represents the Boolean expression when $x_i = 1$, i.e. $f_{xi = 1}$; while the node linked by the 0-edge represents the Boolean expression when $x_i = 0$, i.e. $f_{xi=0}$. The two outgoing edges are given by two functions low(u) and high(u).

Table 1: Truth table of a Boolean Function f

x1	x2	x3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

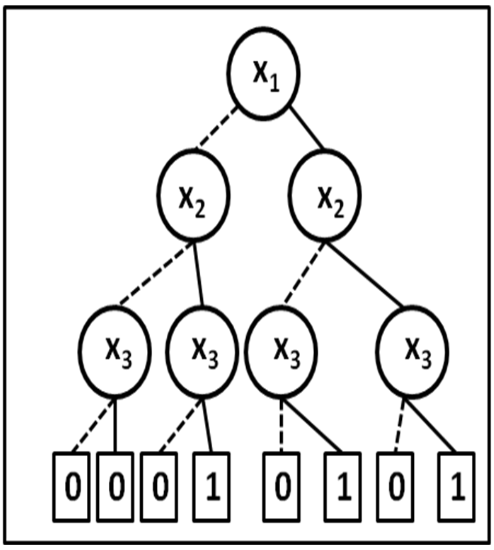

Fig. 1: Decision Tree of the given Boolean Function

Indeed, such representation is space consuming. It is possible to shrink by using following three postulates [30].

Remove Duplicate Terminals : Delete all but one terminal vertex with a given label, and redirect all arcs into the deleted vertices to the remaining one.

Delete Redundant Non Terminals : If non terminal vertices u, and v have var(u) = var(v), low(u) = low(v), and high(u) = high(v), then delete one of the two vertices, and redirect all incoming arcs to the other vertex.

Delete Duplicate tests : If non terminal vertex v has low(v) = high(v), then delete v, and redirect all incoming arcs to low(v).

The shrinking process is shown in figure 2.

Ordered Binary Decision Diagram

For an ordered BDD (OBDD), we impose a total ordering ≺ over the set of variables and require that for any vertex u, and either non terminal child v, their respective variables must be ordered [7].

A. Dual Binary Decision Diagram

If two or more BDD have the same size and representing the same Boolean function, then these BDD are known as Dual BDD, because they are Dual of each other [16].

B. Modified Binary Decision Diagram

The modified binary decision diagram (MBDD) is a binary decision diagram which is either dual BDD or the smaller size BDD. [14, 19].

III. NETWORK RELIABILITY

The reliability of a network G is the probability that G supports a given operation. We distinguish three kinds of operation and hence three kind of reliability.

Two Terminal Reliability

It is the probability that two given vertices, called the source and the sink, can communicate. It is also called the terminal-pair reliability [30].

K Terminal Reliability

When the operation requires only a few vertices, a subset k of N(G), to communicate with each other, this is K terminal reliability [7].

All Terminal Reliability

When the operation requires that each pair of vertices is able to communicate via at least one operational path, this is all terminal reliability. We can see that 2-terminal terminal reliability and all terminal reliability are the particular case of K-terminal reliability [8].

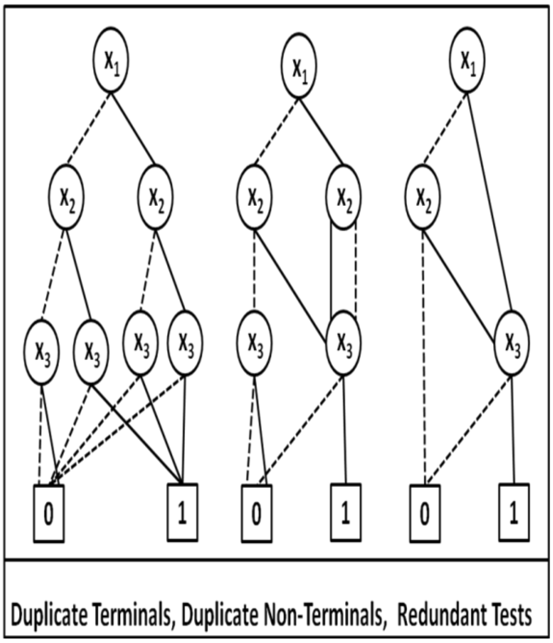

Fig. 2: Reduction Process of the decision tree

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IV. RELIABILITY COMPUTATION OF THE NETWORK

Let us take an example of a directed network represented in the form of a directed graph G (V, E) with single source S and single sink T as shown in figure 3. The network has six nodes and eight edges. The network has four min-paths from source S to sink T.

These are

$H_1 = \{e_1, e_2, e_3\}$, $H_2 = \{e_4, e_5, e_6\}$, $H_3 = \{e_4, e_7, e_2, e_3\}$ and $H_4 = \{e_4, e_5, e_8, e_3\}$

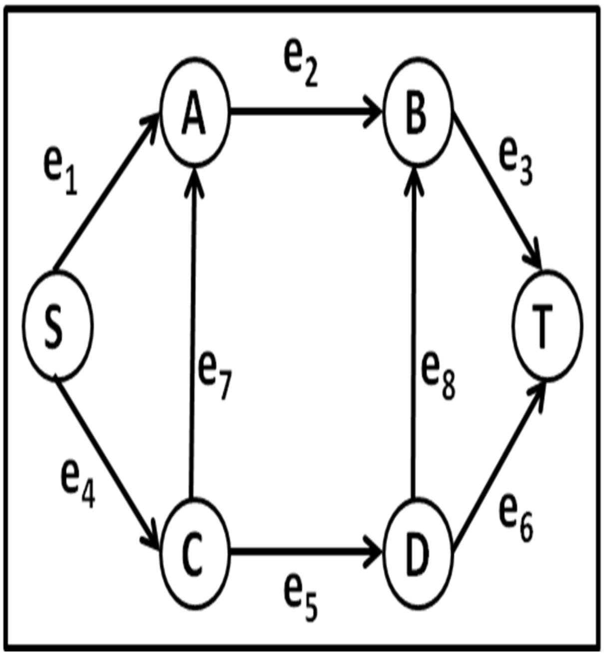

Fig 3: A Directed Network

Let H_1, H_2,-----H_n be the n min-paths from source to sink in a network then the network connectivity function C can be represented as a logical OR of its min-paths.

$C = H_1 U H_2$----UH_i-----UH_n

So the point to point reliability is:

$R_s = Pr\{C\} = Pr\{ H_1 U H_2$----$UH_i$-----$UH_n\}$ --------(1)

So the network connectivity of our network can be expressed as

$C_{1\text{-}6} = e_1e_2e_3 \; U \; e_4e_5e_6 \; U \; e_4e_7e_2e_3 \; U \; e_4e_5e_8e_3$ -------(2)

The probability of the union of non-disjoint events, as in Formula(1), can be computed by several techniques (Exact Methods) [6]. Here we apply the inclusion-exclusion method.

Inclusion-exclusion Method: One method of transforming a Boolean expression Φ(G) into a probability expression is to use Poincare's theorem, also called inclusion-exclusion method [6]. The inclusion-exclusion formula for two minimal paths H_1 and H_2 is express as follows:

$E(H_1 + H_2) = E(H_1) + E(H_2) – E(H_1.H_2)$

Let P_i denote the probability of edge e_i of being working, by applying the Classical inclusion-exclusion formula for calculating the probability of given network (figure 3), we get

$Pr = p_1p_2p_3 + p_4p_5p_6 + p_4p_7p_2p_3 + p_4p_5p_8p_3 – p_1p_2p_3p_4p_5p_6 – p_1p_2p_3p_4p_7 – p_1p_2p_3p_4p_5p_8 – p_2p_3p_4p_5p_6p_7 – p_3p_4p_5p_6p_8 – p_2p_3p_4p_5p_7p_8 + p_1p_2p_3p_4p_5p_6p_7p_8$. -------------- (3)

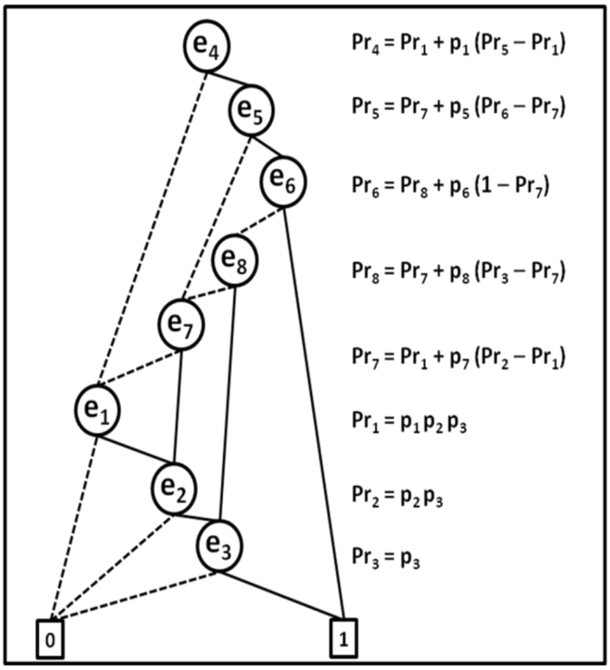

Fig. 4: BDD and its Probability computation

V. GENERATION OF BDD

A particular sequence of variable ordering is known as variable ordering. It has been observed that the size of the BDD strongly depends on the ordering of variables [20]. An ordering is said to be optimal if it generates the minimum size BDD. Friedman [28, 29] has proposed few algorithms to generate optimal variable ordering.The size of the BDD means the total number of non terminal vertices and the number of non terminal vertices at particular level [18]. There are three types of variable ordering (optimal, good and bad) depending on the size of the different BDD [17]. An ordering is said to be optimal if it generates the minimum size BDD. If a directed computer communication network has m disjoints min-paths then these networks are known as disjoint network and m! optimal variable orderings exist to generate the BDD of the given particular CCN [15]. Now we will find the reliability of the given network by using BDD. The BDD based network reliability computation involves three main steps.

(i) Ordering the network edges by using an optimal variable ordering heuristic.
(ii) Generate BDD from the probabilistic graph of the network with the help of network connectivity function.
(iii) Evaluate the network reliability recursively from the BDD by applying Shannon's decomposition.

A. Proposed Heuristic Approach

The heuristic approach is given below:

1. Traverse the graph from source S to sink T. Find all the min-paths from source to sink.

These are $H_1 = \{e_1, e_2, e_3\}$, $H_2 = \{e_4, e_5, e_6\}$, $H_3 = \{e_4, e_7, e_2, e_3\}$ and $H_4 = \{e_4, e_5, e_8, e_3\}$

2. Check whether these paths are disjoint or not. If all the paths are disjoint then we can select any one of the disjoint paths. Then select the second, third and so on.

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3. If all min-paths are not disjoint then find only those min-paths which are disjoint. We have found that the min-paths H_1 and H_2 are disjoint. Now we can move from source S via min-path H_1 or H_2. To choose either H_1 or H_2, we analysis these min-paths. If we select min-path H_1, then the next node is node A. If we select min-path H_2, then the next node is node C.

Since the out degree of the node C is greater than the node A then we will give preference to the higher out degree node. Therefore, we move via min-path H_2 (e_4, e_5, e_6).

Now the question arise that to select the next edge either e_7 or e_8. Since from the edge e_8, we reach to the sink by a single edge, so give preference to the edge e_8 then edge e_7 and then min-path H_1.

After applying the above heuristic approach, we have found the desired ordering.

$\mathbf{e_4 \prec e_5 \prec e_6 \prec e_8 \prec e_7 \prec e_1 \prec e_2 \prec e_3}$.

B. Connectivity Function

The network connectivity function is the union of all min-paths from source to sink, which we have already developed.

$C_{1\text{-}6} = e_1e_2e_3 \cup e_4e_5e_6 \cup e_4e_7e_2e_3 \cup e_4e_5e_8e_3$

C. Shannon's Decomposition

The Shannon's decomposition is defined as follows:

$$\Pr\{F\} = p_1 \Pr\{F_{x_1=1}\} + (1 - p_1)\Pr\{F_{x_1=0}\}$$
$$= \Pr\{F_{x_1=0}\} + p_1(\Pr\{F_{x_1=1}\} - \Pr\{F_{x_1=0}\})$$

where p_1 is the probability of the Boolean variable x_1 to be true and $(1 - p_1)$ is the probability of the Boolean variable x_1 to be false [30].

The BDD and its probability computation for the ordering $\mathbf{e_4 \prec e_5 \prec e_6 \prec e_8 \prec e_7 \prec e_1 \prec e_2 \prec e_3}$ is shown in figure 4.

Since the resulting BDD has only eight non-terminal vertices and there are only eight edges in the given network, so this generated BDD is of minimum size.

Here I have found that the reliability obtained by BDD is equal to the reliability obtained by inclusion-exclusion formula.

$Pr = Pr4 = p_1p_2p_3 + p_4p_5p_6 + p_4p_7p_2p_3 + p_4p_5p_8p_3 - p_1p_2p_3p_4p_5p_6 - p_1p_2p_3p_4p_7 - p_1p_2p_3p_4p_5p_8 - p_2p_3p_4p_5p_6p_7 - p_3p_4p_5p_6p_8 - p_2p_3p_4p_5p_7p_8 + p_1p_2p_3p_4p_5p_6p_7p_8$.

Our program is written in the C language and computations are done by using a Pentium 4 processor with 512 MB of RAM. The computation speed heavily depends on the variables ordering. We found that the reliability obtained by both the methods are same and the size of the BDD is minimum.

VI. CONCLUSION

An alternate and efficient method for generating the BDD of a CCN has been proposed in this paper. I have evaluated the reliability via BDD and by applying an exact method (inclusion-exclusion principle). I have found that the results (reliability) are same by both the methods. I have also found that the size of the BDD (i.e. the total number of nodes and number of nodes in a particular level) is minimal.

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