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# Studies on the Dynamics of a Second Order PLL in the face of Two Input Signals

**Manaj Dandapathak, Bishnu Charan Sarkar**

***Abstract*— The Dynamics of a second order Phase locked loop (PLL) has been critically examined in the face of two co-channel input signals. Applying the analytical tool based on Melnikov's technique, a range of design parameters of the Phase locked loop has been obtained which ensures the stable loop dynamics. It is observed that the said range depends on the relative amplitude and frequency of the input signals. The analytical predictions are verified through numerical simulation results of the system equations.**

***Index Terms: Phase locked loop, Melnikov's function, Voltage control oscillator.***

## I. INTRODUCTION

Phase-locked loop (PLL) systems operating in the stable mode are often used in modern communication systems. Recently there has been an increase of interest in non synchronous PLL operating in chaotic region due to intensive studies of the dynamical chaos and the attempts to apply such PLLs for transmission of information using a chaotic carrier [1-3]. On the co channel the performance of PLL systems in the face of more than one additive input signals is an important problem in the present context as the number of users in the same frequency channel is increasing and due to this the presence of an interfering signal with the desired signal has become a very common event. Experimental and numerical results on the effect of interfering signals on the dynamics of PLL had been reported in the literature [3-6]. But the dynamics of a PLL in the presence of more than one signal has not been studied analytically in details. In this paper attempts have been made to explain the dynamical behavior of the PLL analytically and numerically by solving the system equations of the second order PLL system. To explain the nonlinear dynamical behavior of the system Melnikov's method has been used. By calculating the Melnikov's function, it is possible to predict the values of different parameters for which the system dynamics becomes chaotic [7-10]. In the presence of two additive input signals the system has been mathematically modeled using second order nonlinear differential equation. The phenomenon of chaos has been observed numerically and also by calculating the Lyapunov exponent [10-12] the chaotic behavior has been confirmed. The determination of chaos in the PLL system is important due to two reasons, first one to remove unwanted chaos due to the effect of two input signals which may be appeared to the input of the receiver and secondly by applying external signals of proper strength and frequency, deterministic chaos can be generated in a simple way.

The whole study has been divided into the following sections. In section-II, the system equations have been formulated, in section-III, the system equations have been studied analytically by using Melnikov's method and the analytical results have been discussed, in section-IV, the numerical results are obtained by solving the system equations numerically, and finally in section-V some conclusions are made.

## II. FORMULATION OF SYSTEM EQUATIONS

Block diagram of analog Phase Locked Loop is shown in figure 1, which consists of a Phase Detector (PD), a Low Pass Filter (LPF) and a Voltage Control Oscillator (VCO).

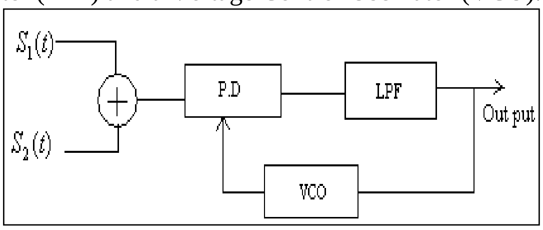


Figure.1 Block diagram of the Phase locked loop

The input signals is composed of two signals $S_1(t)$ and $S_2(t)$ of the following form

$S_1 = A\sin(\omega_i t)$ and $S_1 = mA\sin((\omega_i + \Delta\omega)t + \theta)$ , Where $m$ is the ratio of the amplitudes of the two signals, whose value is very small compared to the amplitude of the input signal and $\Delta\omega$ is difference in frequencies of two signals, which is taken as very small in comparison to the central frequency of VCO. The resultant input signal to the phase detector may be written as

$$S(t) = S_1(t) + S_2(t) = a(t)\sin(\omega_i t + \psi)$$

Where the amplitude is taken as

$$a(t) = \sqrt{1 + m^2 + 2m\cos(\Delta\omega t)} \approx 1 + \frac{m^2}{2} + m\cos(\Delta\omega t)$$

If $\theta_i$ be phase of the input signal and $\theta_f$ be the phase of the output of VCO, then phase error can be written as $\Phi = \theta_i - \theta_f$ .Now considering proportional plus integrating type low pass filter with high frequency gain $F_0$ and time constant $T_1$ , with transfer function

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$F_1(s)=\dfrac{1+F_0 sT_1}{1+sT_1}$, the normalized system equation can be written as

$$\frac{d^2\Phi}{dt^2}+\left[\alpha+F_0(\beta+m\cos(\omega t))\cos\Phi\right]\frac{d\Phi}{dt}-F_0 m\omega\sin(\omega t)\sin\Phi \quad (1)$$
$$+\left[\alpha\beta+m\alpha\cos(\omega t)\right]\sin\Phi=\alpha\Omega_n$$

Where the normalized time is taken as $(Ak)t=t$ and

$\omega=\Delta\omega/Ak$ , $\alpha=1/(AkT_1)$ $\beta=1+\dfrac{m^2}{2}$ and $k$ is total loop gain. Equation (1) is the system equation for PLL system in the presence of two interfering signals. Equation (1) can be written as the set of two 1st order equations as

$$\dot{\Phi}=y \quad (2a)$$

$$\dot{y}=\alpha\Omega_n-\left[\alpha+F_0(\beta+m\cos(\omega t))\cos\Phi\right]y$$
$$+F_0 m\omega\sin(\omega t)\sin\Phi-m\alpha\cos(\omega t)\sin\Phi-\alpha\beta\sin\Phi \quad (2b)$$

By using the equations (2a) and (2b) the dynamics of the PLL system under the effect of two input signals can be explained.

## III. NONLINEAR ANALYSIS USING MELNIKOV'S METHOD

The chaotic behavior of the system can be explained by the analysis of equations (2a) and (2b) in some parameter region via Melnikov's method. This method may be applied to a class of nonlinear system whose system equations can be expressed as the following form

$$\dot{x}=f(x)+\varepsilon g(x,\dot{x},t)$$

Where $g(x,\dot{x},t)$ can be treated as perturbed term and $\varepsilon$ is a small positive number. The unperturbed system can be written as $\dot{x}=f(x)$, which is assumed to have homoclinic orbit associated with a saddle equilibrium point. The existence of homoclinic points can be calculated by calculating the following integral, called the Melnikov function which is defined as [7]

$$M(t_0)=\int_{-\infty}^{\infty} f(q_0(t))\Lambda g(q_0(t),t+t_0)dt$$

Where $q_0(t)$ denotes the homoclinic orbit associated with saddle equilibrium point and $\Lambda$ denotes the wedge product $f_1 g_2-f_2 g_1$. If $M(t_0)$ is bounded away from zero for all $t_0$, there are no homoclinic points. If $M(t_0)$ has a transverse zero or changes its sign, then there exist infinitely many homoclinic points due to the intersection of stable and unstable manifolds, which represent the chaotic behavior of the systems.

Now if $\alpha, F_0, m, \omega, \Omega_n$ are small then for the sinusoidal phase detector the unperturbed system equation can be written as

$$\ddot{\Phi}+\alpha\beta\sin\Phi=0 \quad (3)$$

Solving this equation we have the equations of homoclinic orbit as

$$\Phi=\pm 2\sin^{-1}\left[\tanh(t\sqrt{2\alpha\beta})\right] \quad (4a)$$

$$\dot{\Phi}=\pm 2\sqrt{2\alpha\beta}\,\mathrm{sech}(t\sqrt{2\alpha\beta}) \quad (4b)$$

Using the above two equations the Melnikov's function may be written as

$$M(t_0)=\int_{-\infty}^{\infty}\dot{\Phi}\left[\alpha\Omega_n-\{\alpha+F_0(\beta+m\cos(\omega t))\cos\Phi\}y\right. \quad (5)$$
$$\left. F_0 m\omega\sin(\omega t)\sin\Phi-m\alpha\cos(\omega t)\sin\Phi\ \right]dt$$

Solving the integral by residue method the Melnikov's function may be written as

$$M(t_0)=2\pi\alpha\Omega_n-8\sqrt{2}\alpha^{\frac{3}{2}}\beta^{\frac{1}{2}}-\frac{8\sqrt{2}}{3}F_0\alpha^{\frac{1}{2}}\beta^{\frac{3}{2}}$$
$$+2\pi R\sin(\omega t_0+\theta)\,\mathrm{cosech}\left(\frac{\pi\omega}{4\alpha\beta}\right)\mathrm{sech}\left(\frac{\pi\omega}{4\alpha\beta}\right) \quad (6)$$

Where $R=\sqrt{\dfrac{\omega^2}{\beta^2}+\left[\dfrac{F_0\omega^2}{\alpha\beta}-\dfrac{1}{6}F_0\left(\dfrac{\omega^2}{\alpha\beta}+2\right)\right]^2}$ and

$$\theta=\tan^{-1}\left(\frac{\dfrac{F_0\omega^2}{\alpha\beta}-\dfrac{1}{6}F_0\left(\dfrac{\omega^2}{\alpha\beta}+2\right)}{\omega/\beta}\right)$$

The dynamics of the Phase locked loop under the effect of two additive signals can be understood by the analysis of Melnikov's function for the different value of the system parameters. Depending on the value of the ratio of two signals the system dynamics may be chaotic, if the Melnikov function becomes zero for some value of $t_0$ at which the real solution of the system exists. The minimum value of the ratio of the strength of signals can be calculated from the expression of Melnikov's function. Taking maximum value of $\sin(\omega t_0+\theta)$ as $1$, one can calculate the minimum value of $m$ for which the system may be chaotic from the expression

$$2\pi\alpha\Omega_n-8\sqrt{2}\alpha^{\frac{3}{2}}\beta^{\frac{1}{2}}-\frac{8\sqrt{2}}{3}F_0\alpha^{\frac{1}{2}}\beta^{\frac{3}{2}}$$
$$+2\pi R\sin(\omega t_0+\theta)\,\mathrm{cosech}\left(\frac{\pi\omega}{4\alpha\beta}\right)\mathrm{sech}\left(\frac{\pi\omega}{4\alpha\beta}\right)\geq 0 \quad (7)$$

The perfect value of $m$ can be calculated by observing the phase plane plot and the point at which first transverse intersection is occurred. Plot of Melnikov function with the ratio of the amplitudes of two signals and with the normalized frequency deviation, for $\Omega_n=.8, \alpha=1/7, F_0=0.18$ are shown in figure2 and figure3.

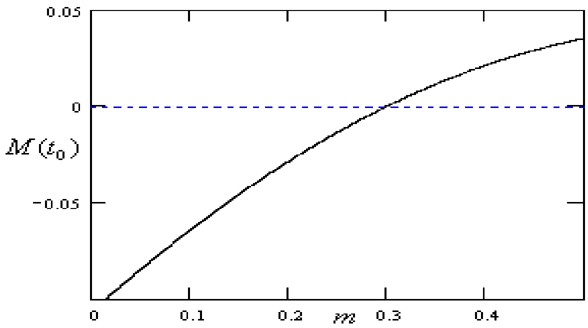


Figure.2 Variation of Melnikov function with the ratio of strength of two signals for $\omega=0.1$



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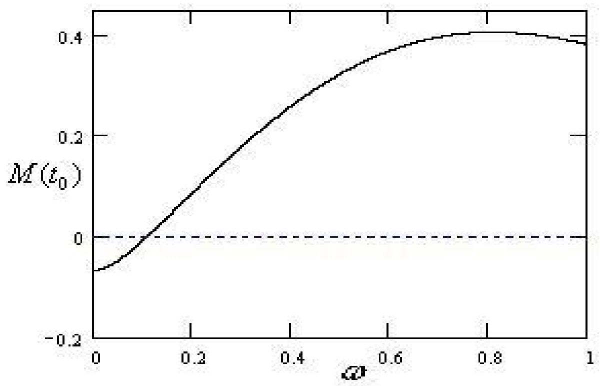


Figure.3 Variation of Melnikov function with normalized frequency deviation for $m = 0.31$

From the plot we see that Melnikov's function becomes zero or changes its sign for some values of $m$ and $\omega$ and for those values the system may exhibit chaotic behavior.

## IV. NUMERICAL RESULTS AND EXPLANATION OF SIMULATION RESULTS BY ANALYTICAL PREDICTION

The system equation (1) has been numerically solved by the Runge-Kutta algorithm by taking $\Omega_n = .8, \alpha = 1/7, F_0 = 0.18., \omega = 0.1$. The simulation results for the phase plane plots for different values of $m$ are shown in figure 4 to figure 7.

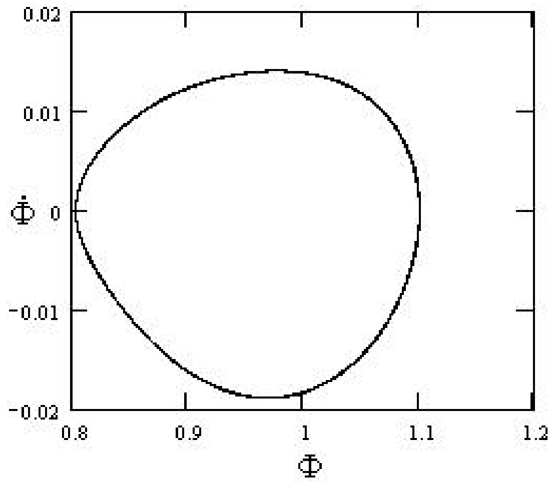


Figure 4. Phase plane plot for $m = 0.1$

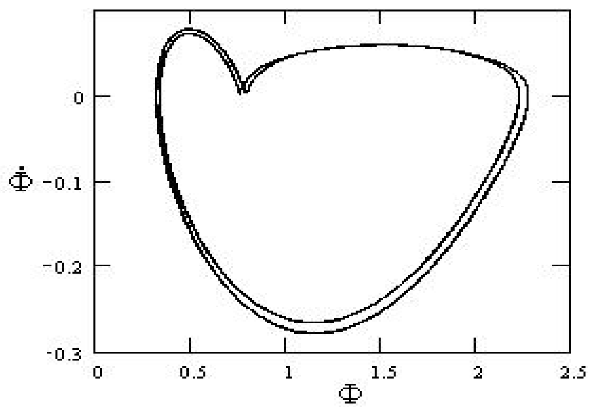


Figure 5. Phase plane plot for $m = 0.30955$

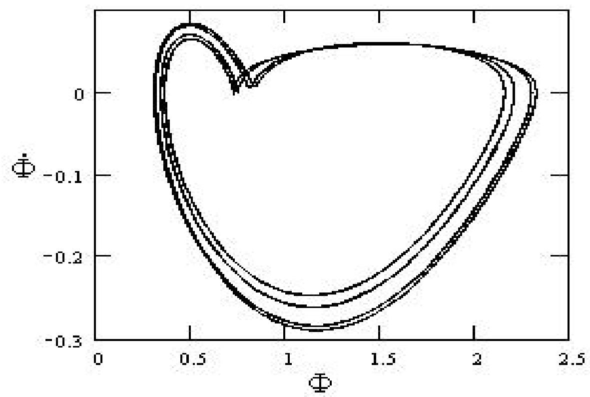


Figure 6. Phase plane plot for $m = 0.3096$

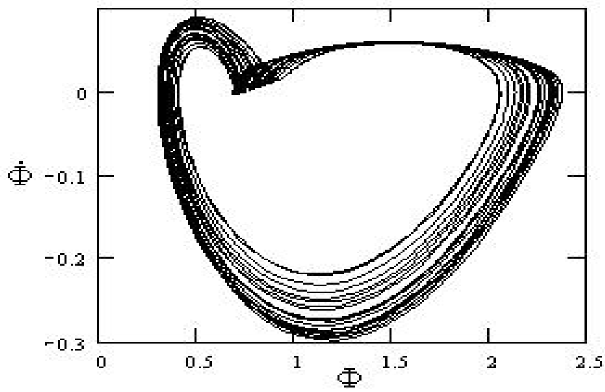


Figure7. Phase plane plot for $m = 0.3098$

From the numerical results we observe that the system dynamics becomes chaotic for some values of the ratio of the strength of the two signals. The small value of $m$ does not affect the system dynamics but with the increase of the ratio it affects the system and for a critical value chaotic oscillation is observed in the system. The presence of chaos can be quantified by computing the Lyapunov exponent numerically for different values of $m$.

Lyapunov exponent is a measure of divergence or convergence of infinitesimally separable trajectories. The hypothesis behind Lyapunov exponents is that there are parameter domains in which the distance between neighbouring solutions exhibits exponential growth in certain direction and exponential decay in another direction. The Lyapunov exponents for the system equations (2a) and (2b) are calculated numerically for $\Omega_n = .8, \alpha = 1/7, F_0 = 0.18., \omega = 0.1$ by the method of Wolf et al. Values of Lyapunov exponents for different values of $m$, the ratio of the strengths of two signals are shown in the table-1.

| Ratio of the strengths of two signals $(m)$ | Lyapunov Exponent-I | Lyapunov Exponent-II |
|---|---|---|
| 0.10 | -0.1680901 | -0.1674086 |
| 0.15 | -0.1672007 | -0.167425 |
| 0.20 | -0.165735 | -0.1673251 |
| 0.25 | -0.1625758 | -0.1654221 |
| 0.30 | -6.2148x10$^{-2}$ | -0.2415397 |
| 0.3095 | 1.08689x10$^{-3}$ | -0.1707812 |
| 0.3096 | 1.08810x10$^{-3}$ | -0.1707812 |
| 0.3098 | 1.10042x10$^{-3}$ | -0.170786 |


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|---|---|---|
| 0.31 | $1.049263\text{x}10^{-3}$ | -0.1707338 |
| 0.32 | $4.330591\text{x}10^{-4}$ | -0.1692999 |

**Table-1**

From the Table-1 it is seen that one of the Lyapunov exponent becomes positive for $m = 0.3095$. So for that value of $m$ or for greater value, the system dynamics may be chaotic. From analytical results as shown in figure 2, we see that when the value of $m$ is near about $0.3$ or greater than that then system dynamics may be chaotic. So we see that numerical results can be explained by analytical prediction. The system dynamics depends on detuning parameter also. In figure 8 we have shown numerically simulated stable and chaotic oscillation regions in $\Omega - m$ parameter space. In figure the zone filled up by red colour represents chaotic oscillation region and the zone filled up by blue colour represents stable region. From figure 8 it is observed that for a fixed value of detuning there is a particular value of $m$ for which the system dynamics is transferred to chaotic state from a stable state.

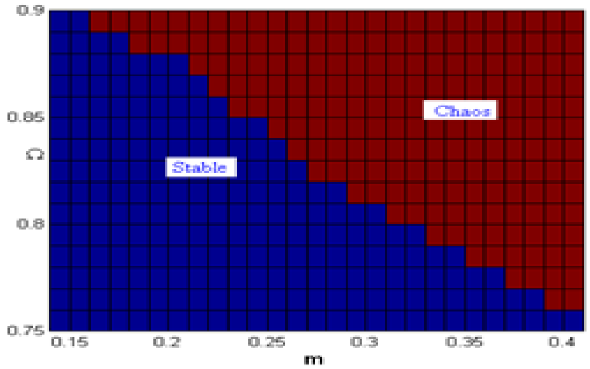


Figure. 8 Numerically simulated Chaotic and stable oscillation regions in $\Omega - m$ parameter space for $\alpha = 1/7, F_0 = 0.18., \omega = 0.1$

Analytically predicted chaotic and stable oscillation regions are shown in figure 9.

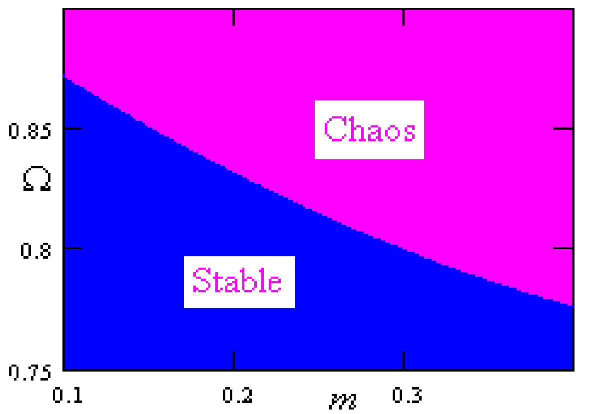


Figure. 9 Analytically predicted Chaotic and stable oscillation regions in $\Omega - m$ parameter space for $\alpha = 1/7, F_0 = 0.18., \omega = 0.1$

From the two results it is observed that analytical predictions are quite similar to numerical simulation results. This similarity shows that the analytical result is applicable to determine the dynamics of a PLL system under the effect of more than one input signals for another set of values of system parameters and strength of second input signal but as Melnikov's method is a perturbation method, the analytical results are not valid for very small or very large values of $m, \omega$ and system parameters.

## V. CONCLUDING REMARKS:

In this paper we study the dynamic regimes of a second order Phase locked loop with two additive input signals. The analytical and numerical analysis of the system equations shows that the dynamics may be chaotic for some value of parameters. Strength of the input signal and frequency range for which the system may be chaotic can be calculated from the analytical results. It is observed that the analytical results closely agree with the numerical results. This study is very helpful in the design of PLL based receivers as the PLL dynamics may be chaotic in the presence of two signals at the input. By proper choice of the system parameters the unwanted chaos can be removed from the system. This study will also be helpful to design the deterministic chaos generators based on PLLs by introducing a suitable second signal at the input of the loop along with normal input. Such systems could be used in chaos based communication system.

## REFERENCES

[1] V.V.Shakhgildyan, L.N.Bellyustina, eds., "Phase locking systems [in Russian]", Radio I Svyaz, Moscow (1982), p-55.

[2] L.M. Pecora and T.L. Carroll, Phys. Rev. Lett, 64, No. 8,821 (1990)

[3] T. Endo, "A Review of Chaos and Nonlinear dynamics in Phase locked loop" IEEE Trans. Circuits & System, Vol.- 331B, No. 6, pp-859-902,1994

[4] B.C. Sarkar and R. Hati, " Chaos from second order PLL in the presence of CW interference", Electron . Lett., vol.35, no,15, pp. 859-902, 1994.

[5] T.Endo, L.O. Chua, "Chaos from phase locked loops" IEEE Trans. Circuits& Systems, vol. CAS-35, no.8, pp. 987-1003,1998.

[6] Harb BA,& Harb AM, "Chaos and bifurcation in third order phase-locked loop", Chaos, Soliton Fractals, Vol.19, pp 667-672, 2004.

[7] M.A.Lieberman and A.J. Lichtenberg, "Regular and Stochastic motion" Springer, Berlin.

[8] P.J.Holmes and Jerrold. E. Marsden, " Melnikov's method and Arnold diffusion for perturbation of integrable Hamiltonian systems", Journal of Math,Phys, Vol. 23(4), pp -669-675, 1982

[9] Simiu E, Melnikov process for stochasticall perturbed, slowly varing oscillator, application to a model of wind driven coastel currents", Journal of Applied Mechanics, Vol-63, pp-429-35, June,1996.

[10] D.W. Joardan and P.Smith, "Nonlinear Ordinary Differential Equations: An Introduction for Scientist and Engineers" 4th edition, Oxford University Press, New York, 2007

[11] R.C. Hilborn, "Chaos and Nonlinear Dynamics", 2nd edition, Oxford University Press, New York, 2000.

[12] S.H. Strogatz, "Nonlinear Dynamics and Chaos", West View Press, 2007

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