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Design of Fixed Structure Optimal Robust Controller Using Genetic Algorithm and Particle Swarm Optimization

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Abstract: In this paper genetic algorithm (GA) has been applied for for the design of the structure specified optimal robust controllers. The controller structure is pre-specified and the controller design problem is posed as constrained nonlinear optimization problem. The parameters of the chosen controller are obtained solving the nonlinear constrained optimization problem. The performance indices which have been used in the design are integral absolute error (IAE), integral square error (ISE), integral time absolute error (ITAE) and integral time square error (ITSE). The constraints are frequency domain performances related with robust stability and disturbance rejection.

Index Terms—Genetic algorithm, fixed structure controller, ISTE,ISE, IAE, nonlinear optimization, optimal robust controller.

I. INTRODUCTION

Due to model uncertainties present in a dynamical system or plant, there is a need to design robust controller. Robust controller provides robustness in the face of uncertainties [1]. Recently, H_∞ -control techniques have found extensive applications for the design of robust controllers [2]-[3]. These techniques make use of H_∞ norm and robustness of the system is achieved in terms of stability and performance. The main disadvantage of the design techniques based on H_∞ -theory is that the order of the controller is high and, therefore, in the normal design, after the design of the controller, some order reduction technique is required [4].

The parameter optimization techniques help in this regard. Parameter optimization methods start with controller structures that are motivated by the ideas from classical, modern or other techniques. What is meant by the controller structure is a system model with one or more parameter values that can be adjusted. The next step in a parameter optimization method is to select an objective function or performance index that gives the quality of performance. After a controller structure, an objective function and some constraints have been specified, the problem can be posed as non-linear optimization problem which can be solved to get the parameters of the controller. The objective function may consist of time domain and/or frequency domain performances expected from the system. In general, this objective function is non-linear, non-differentiable , discontinuous and non-convex in nature. The optimization methods based on calculus will not work. Only search methods can be used. The classical search methods, such as, Nelder-Mead simplex search would only provide local optimal solution [5]. Evolutionary Algorithms guarantee to provide global or near global optimal solution [6]-[9]. The Evolutionary Algorithms (EA) are part of the artificial intelligence and, often, take their operation principles from the natural animal or social behavior.

In this paper, the objective function which has been used in the optimization is indicative of the time domain performance of the system, namely, integral absolute (IAE), integral square error (ISE), integral time square error (ITSE) and integral time absolute error (ITAE). The constraints which have been imposed in the optimization are related with the robust stability and disturbance rejection.

II. OPTIMAL ROBUST CONTROL

While designing the robust controller, the model uncertainty of the plant is explicitly considered, two kinds of model uncertainties: structured and non-structured. Structured model uncertainty or parametric model uncertainty is caused by the parametric modifications of the plant and can be described by the approaches, such as, interval methods [10]. The causes of non-structured model uncertainty are, usually, non-linearities of the plant or modifications of the operating point. This type of the model uncertainties can be represented using H_∞-theory.

The classical methods of the controller design use a nominal model of the plant. The classical measures of the robustness of the system are gain and phase margins. In the robust controller design methods based on the H_∞-theory, a family of the models of the plant is used. A nominal model of the plant and model uncertainty are considered. It is necessary to guarantee the stability of the feedback control system taking into account the model uncertainty. The conditions of the robust stability and disturbance rejection are described using H_∞-norm.

A. Condition for Robust Stability

The plant dynamics is described by the multiplicative model as given by the equation (1). It is assumed that the model uncertainty $W_m(s)$ is stable and bounded, and that no unstable poles of $G_0(s)$ are cancelled in forming $G(s)$ [4].

$$G(s) = G_0(s)[I + \Delta(s)]W_m(s) \tag{1}$$

If the nominal control system (i.e. perturbation in the plant, $\Delta(s) = 0$) is stable with the controller $C(s,k)$ guarantees robust stability of the control system, if and only if, the following condition is satisfied [9]:

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$$\left\| \frac{C(s,k)G_0(s)W_m(s)}{1+C(s,k)G_0(s)} \right\| < 1. \tag{2}$$

B. Condition for Disturbance Rejection

The condition for disturbance rejection indicates that the maximal amplitude of the output variable y(t) caused by means of the disturbance on the plant output, $d_y(t)$, should not exceed a pre-fixed upper bound γ, i.e.

$$\max \frac{\|y\|_2}{\|d_y\|_2} = \left\| \frac{1}{1+C(s,k)G_0(s)} \right\|_\infty \leq \gamma, \tag{3}$$

where, $\gamma \leq 1$ is a design parameter [9].

III. FORMULATION OF OPTIMIZATION PROBLEM FOR CONTROLLER DESIGN

Once the controller structure is chosen, the design problem is to find out the controller parameters to achieve the time domain and frequency domain performances. The related time domain and frequency domain performance requirements are used to formulate the optimization problem solving which controller parameters will be obtained. In the standard nonlinear optimization, the problem is to find out solution, X, by minimizing the performance index or objective function, subject to the constraints. Both or any of the performance index and constraints may be nonlinear. The problem is posed, mathematically, as [5]

$$\underset{X}{Minimize} \; f(X) \tag{4}$$

subject to:

$$g_j(X) \leq 0, \quad j = 1,2,.......,nic \tag{5}$$

$$h_i(X) = 0, \quad i = 1,2,........nec \tag{6}$$

Here, f(X) and $g_j(X)$ ($h_i(X)$) are performance index and inequality (equality) constraints, respectively. The performance indices and constraints, used in the present study are described in sub-sections A and B, respectively.

A. Performance Indices

A performance index is a quantitative measure of the performance of the system. In this paper, following performance indices are used all of which are representative of time domain performances: Integral Square Error (ISE), Integral Absolute Error (IAE), Integral Time Absolute Error (ITAE), and Integral Time Square Error (ITSE) [11].

B. Constraints

The constraints based on the stability robustness and disturbance rejection, as given by the equations (2) and (3), are used in the optimization.

IV. EVOLUTIONARY ALGORITHMS

Three evolutionary algorithms, genetic algorithm (GA), particle swarm optimization (PSO) and differential evolution (DE) have been used for solving optimization problems. These algorithms are briefly discussed in the following sub-sections:

A. Genetic Algorithm (GA)

GA is a stochastic optimization algorithm that was originally motivated by the mechanisms of natural selection and evolutionary genetics. GAs have been proven to be efficient in many different areas, such as image processing, system identification and fuzzy logic controller design [9]. GAs are inherently parallel, because they simultaneously evaluate many points in the parameter space (search space). Considering many points in the search space, GAs would be more likely to converge to the global optimum. In doing so, GAs make use genetic operators: selection, crossover and mutation.

B. Particle Swarm Optimization (PSO)

PSO is a robust stochastic optimization technique based on the movement and cooperation of swarms. It applies the concept of social interaction to problem solving. It uses a number of particles that constitute a swarm moving around in the search space looking for the best solution. Each particle is treated as a point in an N-dimensional space which adjusts its “flying” according to its own flying experience as well as the flying experience of other particles [8].

Every particle keeps track of its co-ordinates in the solution space which are associated with the best solution (fitness) that has been achieved so far by that particle. The value is called the personal best, ‘pbest’. Another best value obtained so far by any particle in the neighborhood of that particle. This value is called global best, ‘gbest’. The basic concept of PSO lies in accelerating each particle towards its ‘pbest’ and the ‘gbest’ locations, with a random weighted acceleration at each time step.

Each particle tries to modify its position using the information such as the current positions, the current velocities, the distance between the current position and the ‘pbest’, the distance between the current position and the ‘gbest’. The mathematical equations for the searching process are

$$v_i^{k+1} = wv_i^k + c_i rand_1(...)(pbest_i - s_i^k) + c_2 rand_2(...)(gbest - s_i^k) \tag{7}$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \tag{8}$$

where, v_i^k is the velocity of the particle i at the iteration k; w is the weighting function; c_1 and c_2 are weighting factors; $rand_1$ and $rand_2$ are the random numbers between 0 and 1; s_i^k is the current position of the particle ‘i’ at iteration ‘k’; $pbest_i$ is pbest of particle ‘i’ and gbest is the best value obtained by any particle so far.

In the above procedure, the maximum velocity $v_{\max}$ determines the resolution of the fitness regions are searched between the present position and target position. If $v_{\max}$ is too high, the particle might fly past good solution. If $v_{\max}$ is too small, the convergence could be slower. According to experience of PSO, $v_{\max}$ takes often 10% to 25% of the dynamic range of the velocity.

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(a) Weighting function

The following weighting function is utilized in the velocity update function :

$$w = w_{\max} - \frac{(w_{\max} - w_{\min}) * iter}{\max iter} \quad (9)$$

Where, $w_{\max}$ and $w_{\min}$ are the initial and final weights; $\max iter$ and iter are the maximum and current iteration numbers, respectively.

(b) Parameters of PSO

In the present simulation work, the following parameters are used: No of particles=10; c_1 and c_2 = 1; $w_{\max}$ =0.9 ; $w_{\min}$ =0.2; $v_{\max}$ =25% of range of parameter; Max number of function evaluations=1000.

V. DESIGN EXAMPLE

The model of the plant, taken from [14], is described by the following transfer function:

$$G_0(s) = \frac{1.8}{s^2(s+2)}. \quad (10)$$

The controller structure, $C(s,k)$, is taken as

$$C(s,k) = k_1 \frac{s^2 + 2k_4 k_5 s + k_5^2}{(s+k_2)(s+k_3)} \quad (11)$$

The vector k of controller parameters is given by $k = [k_1, k_2, k_3, k_4, k_5]$ which is to be obtained solving the optimization problem.

The multiplicative uncertainty $W_m(s)$ is taken as [14]:

$$W_m(s) = \frac{0.1}{s^2 + 0.1s + 10}$$

(12)

The error signal, $E(s)$, assuming the input signal, $R(s)$ is a unit step, is evaluated as follows:

$$E(s) = \frac{1}{1 + C(s,k)G_0(s)} R(s)$$

(13)

The performance indices ISE, ITSE, IAE and ITAE (one at a time) are used for minimization under only one constraint of equation (2). The GA has been used to solve the optimization problem. The controller parameter vector is searched in the following bounds:

$k_1 = [1,1000]; k_2 = [1,100]; k_3 = [1,100]; k_4 = [0.1,5]; k_5 = [0.1,1]$

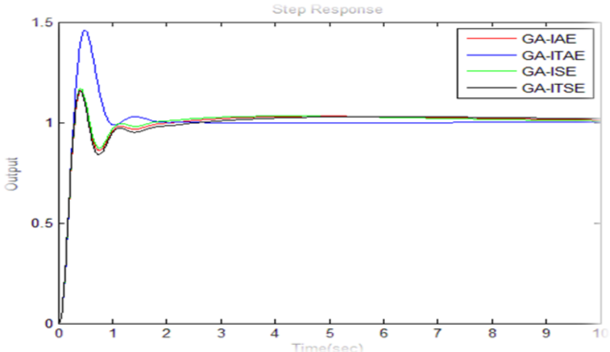

Fig. 1 Step Responses with Controllers Designed using GA

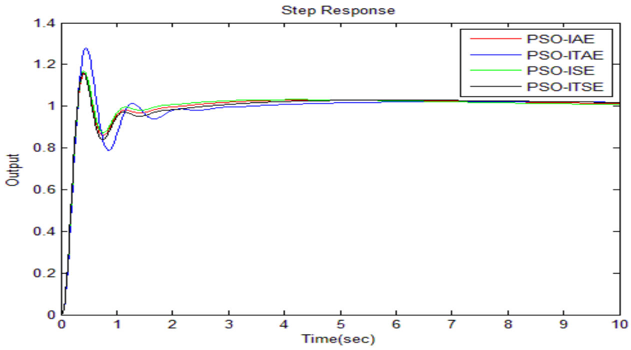

Fig. 2 Step Responses with Controllers Designed using PSO

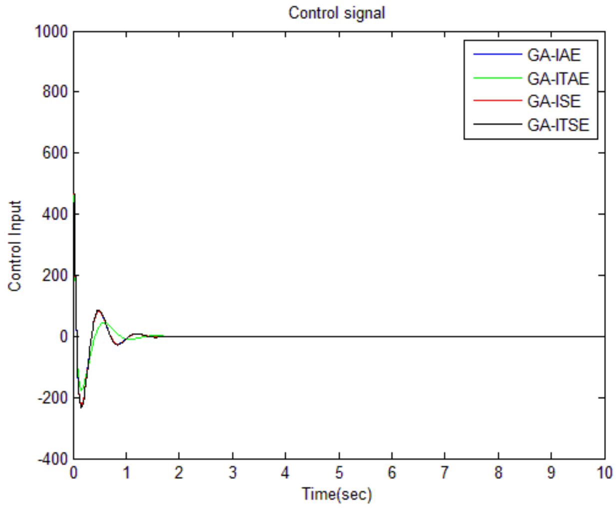

Fig. 3 Control Input at the Plant with Controller Designed using GA

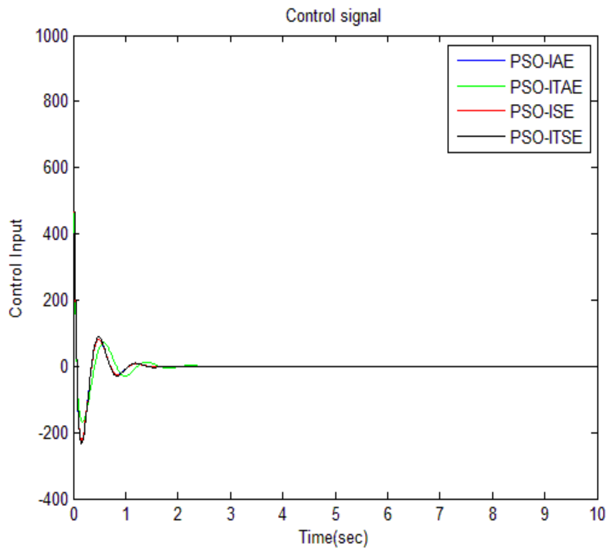

Fig. 4 Control Input at the Plant with Controller Designed using PSO

The Figures 1, 2 show the output of the plant with the controllers designed using GA and PSO for all the four performance indices. The control inputs acting on the plant are shown in the Figures 3,4 with the controllers designed. The overshoot with controller designed using GA is higher for ITAE as compared with controllers designed using PSO for the same performance index.

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In terms of function evaluations for the convergence, GA has taken the higher number for all four performance indices. In all designs, the step response is achieved with zero steady state error. For ITAE, the overshoot with the controller designed using GA is maximum but the control deflection is minimum

The solutions and performances obtained are given in Table I. From the results tabulated the percentage peakovershoot is minimum for the performance index IAE for both GA and PSO. The maximum percentage overshoot of 45.92 is obtained for GA using ITAE performance index, and for PSO it is 27.93 for performance index ITAE also. Another parameter determining the performance is settling time it obtained minimum for GA with ITAE performance index. The maximum value of settling time is rendered by performance index ITSE both with GA and PSO.

TABLE I COMPARISON OF PERFORMANCES OF GA AND PSO FOR DIFFERENT INDICES

Method	k_1	k_2	k_3	k_4	k_5	**Fitness calls**	f_{opt}	e_{ss}	$\% M_p$	t_s **(sec)**	$\|u\|$ **(max)**
GA-IAE	1000	14.03	14.55	1	0.396	9100	7.129	0	15.45	8.17	228.7
GA-ITAE	1000	18.53	15.23	1	1.875	14,100	2.607	0	45.92	1.61	174.9
GA-ISE	1000	14.38	14.25	1	0.448	11,100	3.48	0	16.92	7.18	228.0
GA-ITSE	1000	15.74	12.44	1	0.339	11,100	0.463	0	15.98	9.39	232.9
PSO-IAE	1000	14.21	14.38	1	0.396	4530	7.129	0	15.45	8.17	228.7
PSO-ITAE	1000	4.89	29.88	1	0.264	3930	5.119	0	27.93	9.38	168.4
PSO-ISE	1000	14.32	14.32	1	0.448	2430	3.48	0	16.91	7.17	228.0
PSO-ITSE	1000	13.99	13.99	1	0.340	1530	0.462	0	16.04	9.36	235.2

VI. CONCLUSIONS

The design methodology has been evolved for the optimal robust fixed structure controller by solving the nonlinear optimization problem. The optimization problem so posed has been solved using the GA, PSO. In the design example worked out, only one structure has been taken but the approach is general in nature and can be used for any structure. Moreover, the design facilitates incorporation of many other time and frequency domain performances. Since, in general, complex practical control system designs require many objectives to be met simultaneously along with various constraints, the present design methodology using evolutionary algorithms is best suited.

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