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Dynamics of a Nonlinear Digital Resonator in Free Running and Injection Synchronized Mode: A Simulation Study

Saumen Chakraborty, Bishnu Charan Sarkar

***Abstract*— The structure of a linear digital resonator (DR) has been modified to realize the digital equivalent of Vander Pol Oscillator and the modified system has been found to exhibit several nonlinear dynamical phenomena like synchronization, quasiperiodicity and chaos. Like its linear counterpart, the nonlinear DR can be implemented using common reliable building blocks and so the proposed system can be used as a chaos generator potentially useful in chaos-based communication systems. The dynamics of the nonlinear digital resonator has been studied through numerical simulation.**

***Index Terms*— chaos, nonlinear digital resonator, quasiperiodic , synchronization, Vanderpol oscillator.**

I. INTRODUCTION

Conventional digital resonators (DR) based on adders, multipliers and delay blocks, exhibit natural oscillations at some particular frequency depending on the values of the multiplying factors used to design a DR. DRs are being used as frequency synthesizer in different cases [1]. However, in DR the amplitude of oscillation differ depending upon the initial values and these oscillators cannot be synchronized to the external signals since they are basically linear systems. The study of nonlinear dynamics has brought new excitement in science. It has been well reported that introduction of nonlinearity in various analog and discrete oscillators make them synchronizable to external signals, quasiperiodic, chaotic etc [2]. Vander Pol oscillator is one of such analog system [3-4]. In the present paper, the structure of a linear DR has been modified to realize the digital equivalent of a Vander Pol Oscillator. The modified DR has been found to show several nonlinear phenomena like the synchronization to external signals, quasiperiodicity and non-autonomous chaos etc. The characteristics of these different modes are identified and discussed here.

The paper has been organized in the following way. In section-II, the mathematical model of the system is formulated. Section-III describes the results of numerical simulation study by solving the system equations with different parameter values. In the last section, some conclusions on the importance of the study are presented.

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***** Correspondence Author (s)
Saumen Chakraborty*, Department of Physics, Bidhan Chandra College, Asansol-4, West Bengal, India.
Bishnu Charan Sarkar, Department of Physics, University of Burdwan, Golapbag-4, West Bengal, India.

II. SYSTEM EQUATION FORMULATION

Figure-1 shows the block diagram of a generic digital resonator. The system equation of the DR in terms of design parameters c_1 and c_2 is given by,

$$y[n] = -c_1 y[n-1] - c_2 y[n-2] \quad (1)$$

The frequency transfer function of this resonator in Z domain is,

$$H(z) = \frac{z^2}{z^2 + c_1 z + c_2}$$

(2)

Obviously, a conventional DR is a second order system whose dynamics could be understood finding the poles and zeros of H(z). Here, the poles of the transfer function are,

$$p_{1,2} = -\frac{c_1}{2} \pm \frac{1}{2}\sqrt{c_1^{\ 2} - 4c_2} \quad (3a)$$

Depending on the values c_1 and c_2 the poles $p_{1,} p_2$ can be either real $(c_1^{\ 2} \geq 4c_2)$ or complex $(c_1^{\ 2} \langle 4c_2)$. For real poles the system is equivalent to a serial connection of two first order filters with real co-efficients. But in the second case, two complex conjugate poles could be written as ,

$$(p, p^*) = -\frac{c_1}{2} \pm \frac{j}{2}\sqrt{4c_2 - c_1^{\ 2}} \quad (3b)$$

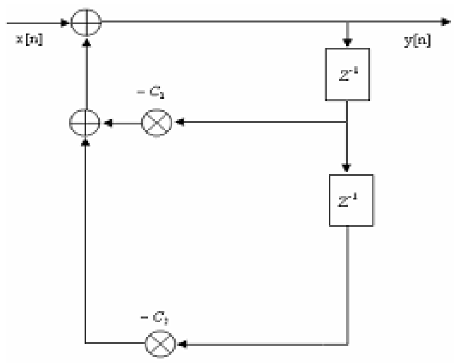

Figure-1: Block diagram of the conventional DR

Where $j = \sqrt{-1}$. In this case the dynamics of the system is a bit complex. If the complex poles of the system are represented by,

$$p = re^{\pm j\omega T}$$

(3c)

Where T stands for the sampling interval of the digital system then one gets the parameter of the DR in terms of r and ωT as,

$$c_1 = -2r\cos\omega T$$

and,

$$c_2 = r^2$$

(3d)

Now the unit impulse response of a system given by (1) with parameter values given by (3d) would be written as,

$$y[n] = \frac{r^n}{\sin\omega T}\sin[(n+1)\omega T] \qquad (4)$$

In this case $y[n]$ has an oscillatory behavior. For a sustained oscillation, one should have $r = 1$. This means the parameters c_1 and c_2 should satisfy following conditions

$$c_1 = -2\cos\omega T \qquad (4a)$$

$$c_2 = 1 \qquad (4b)$$

to make sustained oscillation possible. The parameter c_1 determines the frequency of the oscillatory DR system and its range is given as -2< c_1 <2. But here if we put $c_1 = 2$, the poles become real (from (3b)) and the oscillatory behavior of the system would be absent. So, to have oscillation c_1 must be less than 2 or that is to have sustained the oscillatory behavior, $c_1 - c_2 < 1$.

An external sinusoidal signal of frequency close to that of the DR is applied at the input of the DR to synchronize it. As expected no synchronization of the DR could be observed, since it is a linear system. It is well known that the presence of nonlinearity of some form in a system is necessary to synchronize it as well as to get nonlinear response of the oscillating system. Here we would introduce a nonlinearity equivalent to the one found in Vander Pol oscillator.[3]

Vander Pol type Nonlinear Digital Resonator: A Vander Pol oscillating system equation is obtained by adding a nonlinear term $-\varepsilon(1-y^2)\frac{dy}{dt}$ to the equation representing undamped simple harmonic oscillation. Following this, we add a discrete equivalent of the above term to the conventional DR.

We want to get a discrete time domain equation equivalent to

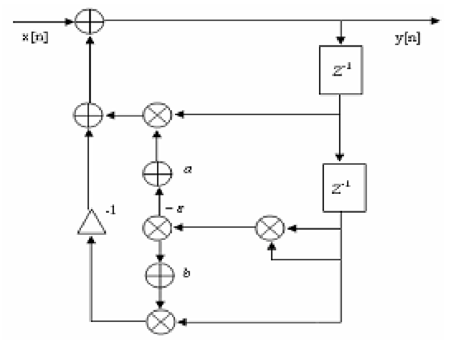

Figure-2: Block diagram of the modified system

Vander Pol equation which would represent a discrete time Vander Pol oscillator showing oscillation building up phenomenon as well as the synchronization to external signals. $\frac{dy}{dt}$ can be approximated by the following equation,

$$\frac{dy}{dt} = y[n-1] - \beta y[n-2]$$

Where, β is a parameter representing inevitable loss in practical system. Then the equivalent difference equation of the system under consideration becomes,

$$\begin{aligned} y[n] &= (c_1 + \varepsilon - \varepsilon y^2[n-2])\,y[n-1] \\ &- (c_2 + \varepsilon\beta - \varepsilon\beta y^2[n-2])\,y[n-2] \end{aligned} \qquad (5)$$

If the damping is low enough then we consider $\beta = 1$, for which we get

$$\begin{aligned} y[n] &= (a - \varepsilon y^2[n-2])\,y[n-1] \\ &- (b - \varepsilon y^2[n-2])\,y[n-2] \end{aligned}$$

(6)

Where, $a = c_1 + \varepsilon$

$b = c_2 + \varepsilon$

(7)

Here it is easily noted that a must be greater than c_1 by an amount ε and b is also greater than c_2 by an amount ε i.e. b is always greater than 1. From equation (7) we observe here also $c_1 - c_2 < 1$ as said before to make oscillation possible (for lower ε values).The block diagram of equation (6) is given in Figure-2. If we put $\varepsilon = 0$, the system reduces to a linear digital resonator represented by (1).

III. Simulation Study

The time domain response of the system is examined with the help of equation (6). We get Fig-3 which simply illustrates the oscillation building up phenomenon of the discrete time Vander Pol oscillator. In the said system on increasing ε the amplitude decreases. When ε low enough, the limit cycle is near circular, for higher value of ε the forms of the limit cycles vary from the most circular curve to the curviest as shown in Fig -4.

It becomes too difficult to calculate the natural frequency of the system due to the presence of cubic term in the equation, but on simulation we observe that for lower value of ε (<0.01) the natural frequency of system remains almost same to that of linear one i.e.

$$\omega_0 = \frac{1}{T}\cos^{-1}(-c_1/2\sqrt{c_2}) \cong \frac{1}{T}\cos^{-1}(-a/2\sqrt{b}).$$

To study the possibility of synchronization of the proposed nonlinear DR, we examine (6) with a forcing external signal given by $d\sin(n\omega_0 k)$. Here d is the strength of the external signal and k represents the ratio of the frequency of the external signal to that of the natural frequency of the DR. The equation to be studied to get the forced response of the system is:

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$$y[n] = (a - \varepsilon y^2[n-2])y[n-1] - (b - \varepsilon y^2[n-2])y[n-2] + d\sin(n\omega_0 k) \quad (8)$$

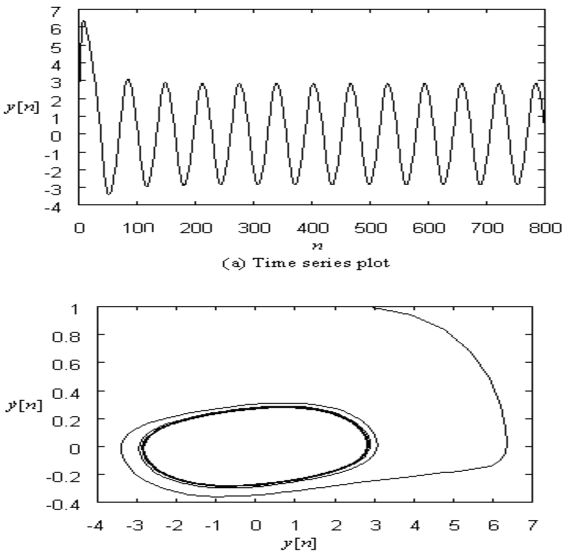

Figure-3 : Oscillation building up phenomenon with $a = 2.01, \varepsilon = 0.01, b = 1.02$

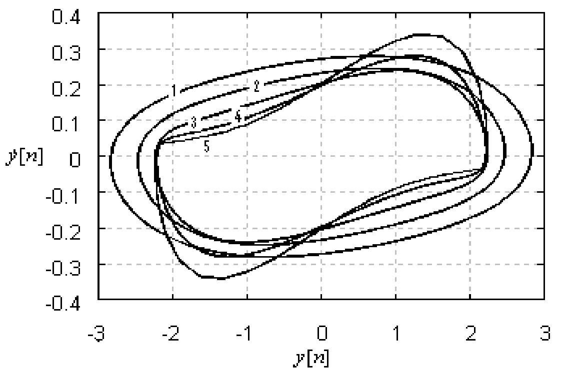

Figure-4: Limit cycles for discrete system with $[a, \varepsilon] = [2.01, .01]^1; [2.02, .02]^2; [2.05, .05]^3; [2.1, .1]^4; [2.15, .15]^5$

The natural frequency (ω_0) of the system is determined in terms of n .With $a = 2.0001, \varepsilon = 0.0001, b = 1.0101$ the natural frequency $\omega_0 = 0.01572$ (as $\omega = 1/n$). In Fig-5 we plot the time series input, output and input versus output for $d = 0.1$ $k = 1.2$ i.e. frequency of the external signal $\omega = 0.01886$. The output signal obtained from the DR shows ten cycles for n = 530, therefore frequency of the output is $10/530 = 0.01886$. This means a synchronization of the DR has taken place with the injected signal. But from Fig-5(c) we see there is a phase difference between input and output. Fig-7 shows that, the synchronization range increases with increase of the strength of forcing signal and with increase of the value of nonlinear parameter i.e. ε .

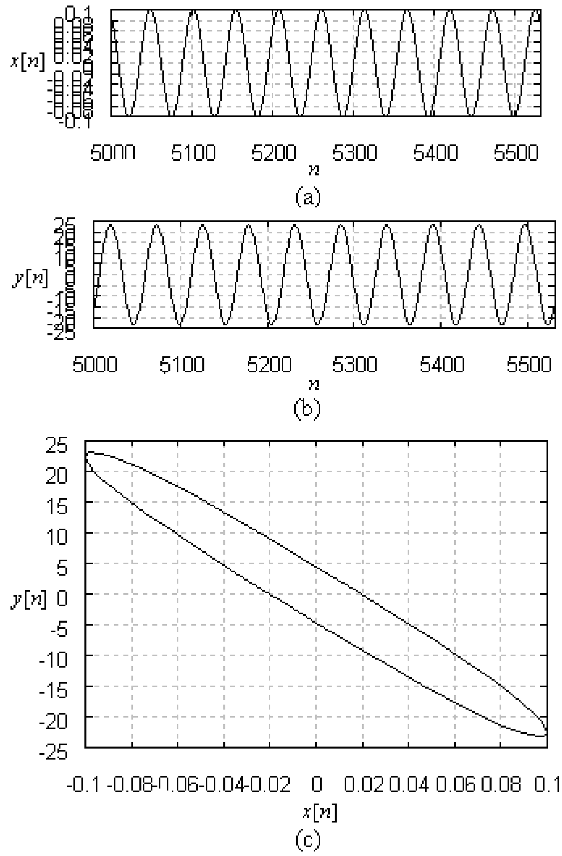

Figure -5: (a) Time series of external signal. (b) Time series of output. (c) External input signal vs. Output of the system with $d = .1, k = 1.2, a = 2.0001, \varepsilon = 0.0001, b = 1.01$

A related phenomenon to synchronization is quasiperiodicity. This occurs when the amplitude of the forcing function is fairly low. The natural frequency (ω_0) of the system competes with the forcing frequency in determining the frequency of the system and the result is that the frequency and amplitude varies with time. The phase plane trajectory is confined to an annular region (Fig-6).

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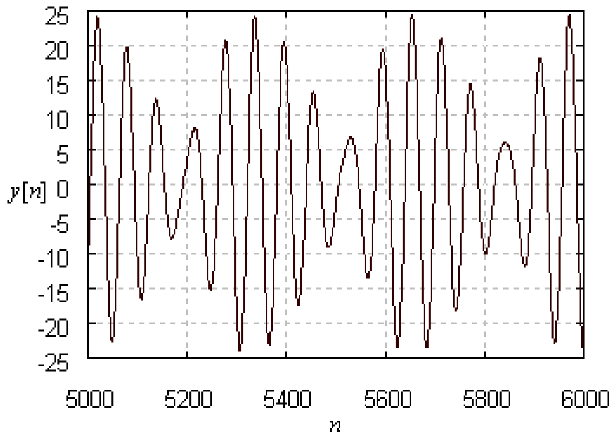
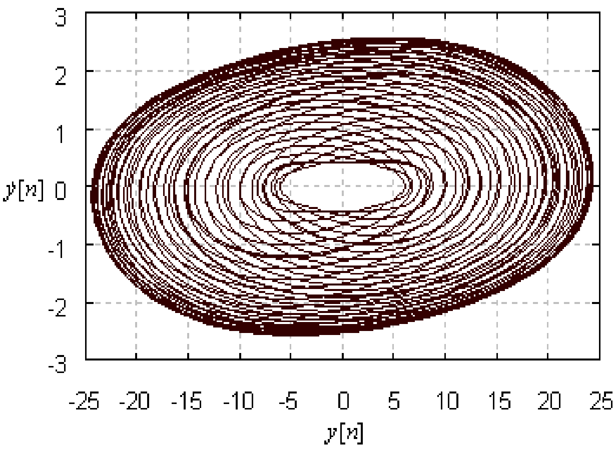


(a) Time series plot



(b) Phase plane plot

Figure-6: The quasiperiodicity of the system with $d = .04, k = 1.2$ ($a = 2.0001, \varepsilon = 0.0001, b = 1.01$)

On simulation it is observed that as the value of nonlinear parameter of the system increases, the quasiperiodicity occur at the lower strength of the forcing signal i.e. the system is synchronized by weaker sync signal due the presence of the greater amount of nonlinearity in the system(Fig-8).

With $a = 2.11, \varepsilon = 0.11, b = 1.12$ if we increase the strength of forcing signal the steady state behavior shows a bifurcation and goes to period-2 oscillation for finite time duration. Further increase in strength results in period-4 oscillation and ultimately the system becomes chaotic (Fig-9). Between the chaotic behaviors a period-3 is observed, which on increase of strength through bifurcation again becomes chaotic. Again, all these phenomena are obtained by varying the nonlinear parameter (ε) keeping the strength of the forcing signal fixed as focused in Fig-10. Fig-11 shows that if the system parameter and the forcing signal strength remains fixed with the variation of frequency factor the system goes to chaos trough bifurcation and then again return to single state in similar fashion.

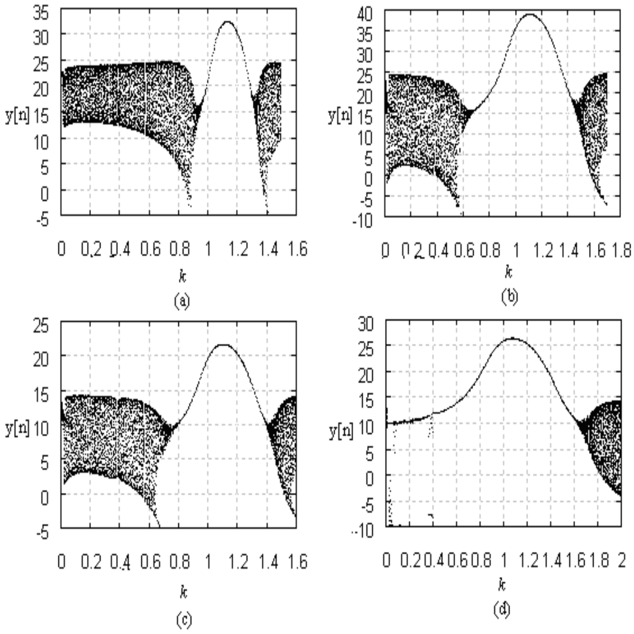
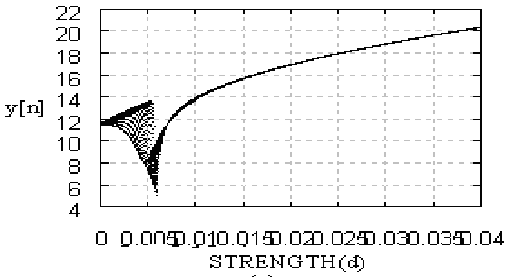
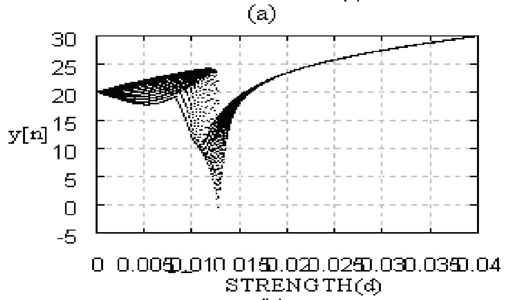


Figure-7: Synchronization range depends on nonlinearity as well as the strength of the forcing signal is shown (a),(b) with $\varepsilon = 0.0001$ & (c),(d) with $\varepsilon = 0.0003$ and (a),(c) with $d = 0.05$ &(b),(d) with $d = 0.1$



(a)



(b)

Figure-8: Quasiperiodic range decreases with the increase of nonlinearity in (a) $\varepsilon = 0.0003$ and (b) $\varepsilon = 0.0001$

IV. CONCLUSION

Introduction of non-linearity makes the DR synchronizable as well as it can be easily designed with reliable design components. Study of non linear dynamics of proposed DR in the face of external forcing signal has been carried out. Results show synchronization, period-2, period-4, chaos all the non linear dynamics which are consistent with the result obtained in conventional analog oscillators. Thus, the system can be used in chaos generation, such as chaos moderator etc. Vander Pol type nonlinearity makes the system dynamics more complex and to solve it analytically may be a challenging problem for researchers. In future the experimental studies are necessary for implementation; it may also be used in chaos synchronization communication.

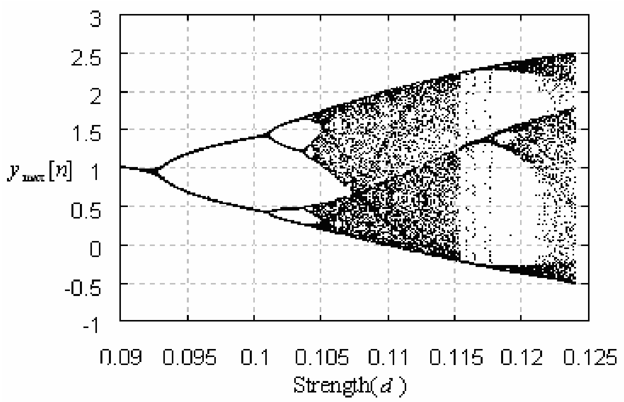


Figure-9: Shows variation of the output of the modified system with increasing strength with $k = 2.28 (a = 2.11, \varepsilon = 0.11, b = 1.12)$.

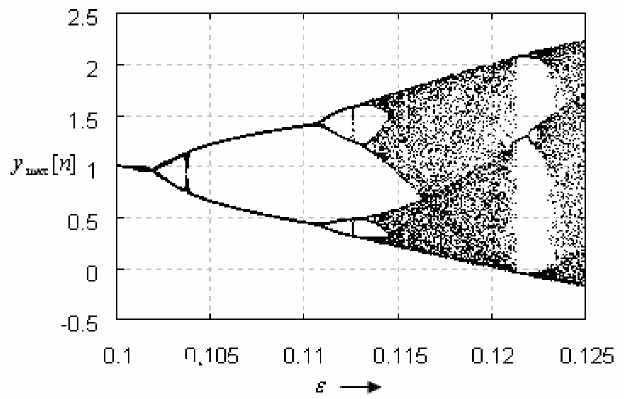


Figure-10: Shows variation of the output of the modified system with increasing nonlinearity.

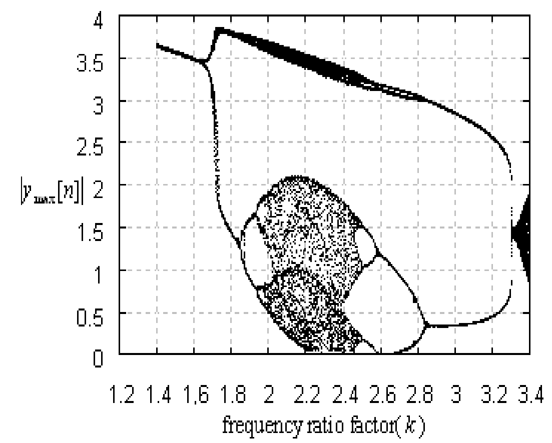


Figure-11: Shows variation of the output of the modified system with increasing frequency with $d = 0.11 (a = 2.11, \varepsilon = 0.11, b = 1.12)$

REFERENCES

[1] John G. Proakis, Dimitris G. Manolakis; Digital Signal Processing Principles, Algorithms, and Applications; Prentice Hall of India Pvt. Ltd., New Delhi, 2004(3rd Ed)

[2] Almudena Suarez , Raymond Quere; Stability Analysis of Nonlinear Microwave Circuits; Artech House, Boston, London, 2003

[3] Robert C. Hilborn, Chaos and Nonlinear Dynamics; Oxford University Press, New Delhi, 2000(2nd Ed)

[4] J Dixon, E Bradley, Z.B. Popovic, Nonlinear time-domain analysis of injection-locked microwave MESFET oscillators; Microwave Theory and Techniques ,IEEE Transactions on Volume45, Issue7, Page(s): 1050–1057, Jul 1997.

Mr. Saumen Chakraborty received his M Sc degree in Physics (Special paper: Radio Physics and Electronics) from Burdwan University in the year 2003 securing first class. He is working as an Assistant Professor in Physics at Bidhan Chandra College, Asansol, W.B. His research areas are nonlinear dynamics of digital and analog electronic oscillators, phase locked loops etc. He is doing Ph.D work under the supervision of Prof. B.C.Sarkar. He has published several international and national conference papers. He is an associate member of IETE, India.

Prof. Bishnu Charan Sarkar received his M Sc degree in Physics (Special paper: Radio Physics and Electronics) from Burdwan University in the year 1975 securing first position in the first class and was awarded with the University Gold Medal. He worked in the field of synchronous communication system and got his Ph D degree in science from Burdwan University in the year 1985. He joined the post graduate faculty of Physics in the University of Burdwan as a lecturer in 1980 and presently working as a Professor in the same department. He is the in-charge of the electronics section and is the coordinator of the M Tech program in ECE (Microwave) of Physics department.

Prof Sarkar in engaged in research on the problems of analog and digital signal processing, synchronous receiver design, synchronization of microwave oscillators, nonlinear dynamics of electronic oscillators and phase locked loops etc. He has supervised the Ph D work of seven students and at present five other students are working under him for their Ph D degrees. He has published about 80 research papers in inland and foreign journals and has contributed more than 80 technical papers in national and international seminars and conferences. Prof Sarkar received K S Krishnan memorial award from IETE (India) and he was a visiting fellow of the CSIR (India). He acts as a reviewer of IEEE (USA), IEE (UK), IET (UK), IJE (UK) etc. Prof Sarkar has written 3 books on computer programming and has edited three books published by different departments of the University of Burdwan. He is presently the chairman of the Burdwan sub-center of the IETE, India.

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