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Comparison of BER Performance in OFDM Using Different Equalization Techniques

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Abstract: The effects of frequency-selective channel conditions, for example fading caused by multipath propagation, can be considered as constant (flat) over an OFDM sub-channel if the sub-channel is sufficiently narrow-banded (i.e., if the number of sub-channels is sufficiently large). This makes frequency domain equalization possible at the receiver, which is far simpler than the time-domain equalization used in conventional single-carrier modulation. In OFDM, the equalizer only has to multiply each detected sub-carrier (each Fourier coefficient) in each OFDM symbol by a constant complex number, or a rarely changed value.

Some of the sub-carriers in some of the OFDM symbols may carry pilot signals for measurement of the channel conditions (i.e., the equalizer gain and phase shift for each sub-carrier). Pilot signals and training symbols (preambles).

Here we modelled OFDM system with equalizers. Two different equalizers, namely Zero Forcing (ZF) and Minimum Mean Square Error (MMSE), along with different tapping are used. The modulation with multicarrier is employed, which provides advantages like inter symbol interference (ISI) reduction, high reliability, and better performance in multi-path fading. These equalizers are adopted to remove the ISI generated in the transmitted data under various fading environments. The results show that, with MMSE and ZFE equalizers, the bit error rate (BER) performance is improved. Further, the BER performance of MMSE is superior to ZFE equalizer.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM), multipath propagation, fading channel, inter symbol interference (ISI).

I. INTRODUCTION

OFDM was initially used for wired and stationary wireless communications. However, with an increasing number of applications operating in highly mobile environments, the effect of dispersive fading caused by a combination of multi-path propagation and doppler shift is more significant.

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multi-carrier modulation technique, which uses several orthogonal sub-carriers to transmit/receive a high data rate signal. It has become an increasingly popular scheme in modern digital communications and is already being applied in DAB, DVB-T, DVB-H, DVB-SH, WiFi, WiMAX and 3GPP LTE wireless standards. The primary advantage of OFDM over single-carrier transmissions is its robustness against frequency-selective fading in a multipath channel, thereby eliminating the need for complex time-domain equalization.

Improvements in FIR equalization using FFTs or partial FFTs leads mathematically closer to OFDM, but the OFDM technique is easier to understand and implement, and the sub-channels can be independently adapted in other ways than varying equalization coefficients, such as switching between different QAM constellation patterns and error-correction schemes to match individual sub-channel noise and interference characteristics.

The original standard family IEEE 802.11 was defined in 1997; and the IEEE 802.11a was defined in 1999. Since then, improvements had been proposed and adopted; and the standard was updated in 2002 to create a standard technology that could span multiple physical encoding types, frequencies and applications. The IEEE committee intends to setup the IEEE 802.11a in the same way as that of the popular IEEE 802.3 Ethernet standard, which has been successfully applied to 10, 100 and 1000 Mbps technology over fiber and various kinds of copper.

The OFDM physical layer (PHY) operates at a carrier frequency of 5GHz in the industrial, scientific, and medical (ISM) frequency bands. The radio frequency for the IEEE 802.11a OFDM layer is initially falling into the three 100MHz unlicensed national information infrastructure (U-NII) bands, 5.15-5.25, 5.25-5.35 and 5.725-5.825 GHz. The centers of the outmost channels shall be at a distance of 30MHz from the band's edges for the lower and middle U-NII bands, and 20MHz for the upper U-NII band. The spectrum allocation is subject to the authorities responsible for geographic specific regulatory domains. In Canada, it is regulated by the License Exempt Local Area Network Code (LELAN). Table 1.1 shows the channel allocation scheme for this standard. There are twelve 20MHz channels, and each band has a different output power limit. The first 100 MHz in the lower section is restricted to a maximum power output of 40mW. The second 100MHz has a higher limit of 200mW, while the top 100MHz is dedicated for outdoor applications, with a maximum of 800mW power output.

Table 1.1 OFDM operating bands and Channels

Band	Channel numbers	Frequency (MHz)	Maximum output power (Upto 6 dB antenna gain)
U-NII lower band 5.15 to 5.25 MHz	36	5180	40mW (2.5mW/ MHz)
	40	5200	
	44	5220	
	48	5240	

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U-NII middle band 5.25 to 5.35 MHz	52	5260	200mW (12.5mW/ MHz)
	56	5280	
	60	5300	
	64	5320	
U-NII upper band 5.725 to 5.825 MHz	149	5745	800mW (50mW/ MHz)
	153	5765	
	157	5785	
	161	5805	

The IEEE 802.11a standard requires the receivers to have a minimum sensitivity ranging from -82 to 65dBm, depending on the chosen data rate shown in Table (1.2). The IEEE 802.11a OFDM system can provide a variable data transmission rate of 6, 9, 12, 18, 24, 36, 48 and 54Mbps. Among these, the support of transmitting and receiving data rate of 6, 12 and 24Mbps is mandatory. The IEEE 802.11a system uses a training sequence for its synchronization. The training sequence comprises of 10 short symbols and 2 long symbols with the total training length of 16ms. The sub-carriers are modulated using binary phase shift keying (BPSK), quadrature PSK (QPSK), 16-quadrature amplitude modulation (16-QAM), or 64-QAM, depending on the data transmission rate.

Table 1.2 Receiver performance Requirements

Data rate (Mbps)	Minimum Sensitivity	Adjacent channel rejection (dB)	Alternate adjacent channel rejection (dB)
6	-82	16	32
9	-81	15	31
12	-79	13	29
18	-77	11	27
24	-74	8	24
36	-70	4	20
48	-66	0	16
54	-65	-1	15

II. OFDM SYSTEM MODEL

A top-level block diagram of the base-band high rate OFDM system is shown in Fig. 2.1.

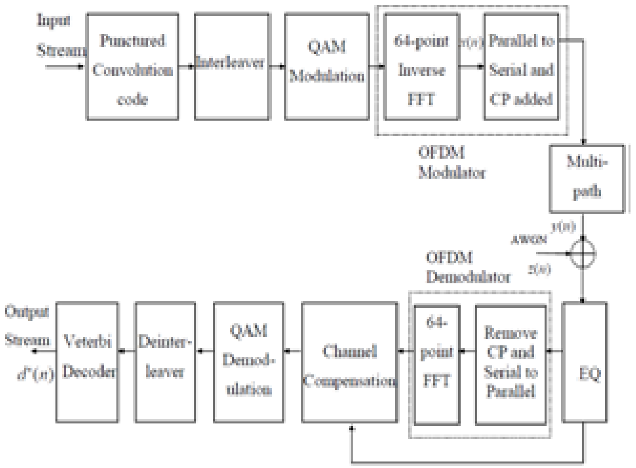

Fig. (2.1) A Block Diagram of the Base-band OFDM

This model is based on the parameters defined in the IEEE 802.11a standard and includes the TEQ and the effective channel compensation function blocks. The function blocks shown in the diagram will be discussed in detail.

From the diagram the data flow can be described as follows. When the data transmission rate is 48 Mbps or 54 Mbps, the punctured convolution code with coding rate R = 2 / 3 or 3/ 4 is adopted respectively. The input stream is first fed into the punctured convolution encoder. The coded bit stream is buffered and block interleaved. After that the binary bits are mapped into QAM signals according to the QAM constellation map. These complex numbers are then buffered to a multiplication of 64 samples, employs a 64-point IFFT operation to generate an OFDM symbol. The output data is then converted from parallel version to serial data, and the cyclic prefix is added. The block inside the dotted line on the upper branch realizes the OFDM modulation. The serial data stream is fed into the multi-path fading channel with additive white Gaussian noise (AWGN). At the receiver the inverse operations are employed. The corrupted signal is first passed to the TEQ finite impulse response (FIR) filter.

The output signal is then converted to the parallel version after discarding the interfered cyclic prefix. A 64-point FFT is used to transfer the signal back to the base band frequency domain. The OFDM demodulator is also indicated in the dotted line box in the diagram (lower branch). Then the effective channel is compensated. After QAM demodulation, de/interleaving, Veterbi decoding, the approximated signal d'(n) is recovered.

III. EQUALIZER

Equalization compensates for inter-symbol interference (ISI) created by multipath within time dispersive channels. If the modulation bandwidth exceeds the coherence bandwidth of the radio channel, ISI occurs and modulation pulses are spread in time. An equalizer within a receiver compensates for the average range of expected channel amplitude and delay characteristics.

Equalization techniques can be subdivided into two general categories- linear and nonlinear equalization. These categories are determined from how the output of an adaptive equalizer is used for subsequent control (feedback) of the equalizer. In general, the analog signal d(t) is processed by the decision making device in the receiver. The decision maker determines the value of the digital data bit being received and applies a slicing or thresholding operation (a nonlinear operation) in order to determine the value of d(t). If d(t) is not used in the feedback path to adapt the equalizer, the equalization is linear. On the other hand, if d(t) is fed back to change the subsequent outputs of the equalizer, the equalization is nonlinear.

(A) Zero-Forcing Equalizer

First, let us consider the use of a linear equalizer, i.e., we employ an LTI filter with transfer function $H_E(Z)$ as the equalizing circuit. The simplest way to remove the ISI is to choose $H_E(Z)$ so that the output of the equalizer gives back the information sequence, i.e. $\hat{I}_k = I_k$ for all k if noise is not present. This can be achieved by simply setting the transfer function $H_E(Z)$ = 1/G(z). This method is called zero-forcing equalization since the ISI component at the equalizer output is forced to zero.

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We note that the effect of the equalizing filter on the noise is neglected in the development of the zero-forcing equalizer above. In reality, noise is always present. Although the ISI component is forced to zero, there may be a chance that the equalizing filter will greatly enhancing the noise power and hence the error performance of the resulting receiver will still be poor. To see this, let us evaluate the signal-to-noise ratio at the output of the zero-forcing equalizer when the transmission filter $H_T(f)$ is fixed and the matched filter is used as the receiving filter, i.e.,

$$H_R(f) = H_T^*(f) H_C^*(f) \qquad (3.1)$$

In this case, it is easy to see that the digital filter $H(Z)$ is given by

$$H\left(e^{j2\pi fT}\right) = \frac{1}{T}\sum_{n=-\infty}^{\infty}\left|H_T\left(f-\frac{n}{T}\right)H_C\left(f-\frac{n}{T}\right)\right|^2 \qquad (3.2)$$

and the PSD of the colored Gaussian noise samples n_k

$$\Phi_{n_k}\left(e^{j2\pi fT}\right) = \frac{N_0}{2T}\sum_{n=-\infty}^{\infty}\left|H_T\left(f-\frac{n}{T}\right)H_C\left(f-\frac{n}{T}\right)\right|^2 \qquad (3.3)$$

Hence, the noise-whitening filter $H_W(z)$ can be chosen as

$$H_W\left(e^{j2\pi fT}\right) = \frac{1}{\sqrt{H\left(e^{j2\pi fT}\right)}} \qquad (3.4)$$

and then the PSD of the whitened-noise samples $\tilde{n}_k$ is simply $N_0 = 2$. As a result, the overall digital filter G(z).

$$G\left(e^{j2\pi fT}\right) = H\left(e^{j2\pi fT}\right)H_W\left(e^{j2\pi fT}\right) = \sqrt{H\left(e^{j2\pi fT}\right)} \qquad (3.5)$$

Now, we choose the zero-forcing filter $H_E(z)$ as

$$H_E\left(e^{j2\pi fT}\right) = \frac{1}{G\left(e^{j2\pi fT}\right)} = \frac{1}{\sqrt{H\left(e^{j2\pi fT}\right)}} \qquad (3.6)$$

Since the zero-forcing filter simply inverts the effect of the channel on the original information symbols I_k, the signal component at its output should be exactly I_k. If we model the I_k as IInd random variables with zero mean and unit variance, then the PSD of the signal component is 1 and hence the signal energy at the output of the equalizer is just $\int_{-1/2T}^{1/2T} df = 1/T$. On the other hand, the PSD of the noise component at the output of the equalizer is $\frac{N_0}{2}\left|H_E\left(e^{j2\pi fT}\right)\right|^2$. Hence the noise energy at the equalizer output is $\int_{-1/2T}^{1/2T}\frac{N_0}{2}\left|H_E\left(e^{j2\pi fT}\right)\right|^2 df$. Defining the SNR as the ratio of the signal energy to the noise energy, we have

$$SNR = \left\{\frac{N_0 T^2}{2}\int_{-1/2T}^{1/2T}\left[\sum_{n=-\infty}^{\infty}\left|H_T\left(f-\frac{n}{T}\right)H_C\left(f-\frac{n}{T}\right)\right|^2\right]^{-1} df\right\}^{-1} \qquad (3.7)$$

Notice that the SNR depends on the folded spectrum of the signal component at the input of the receiver. If there is a certain region in the folded spectrum with very small magnitude, then the SNR can be very poor.

(B) MMSE Equalizer

The zero-forcing equalizer, although removes ISI, may not give the best error performance for the communication system because it does not take into account noises in the system. A different equalizer that takes noises into account is the minimum mean square error (MMSE) equalizer. It is based on the mean square error (MSE) criterion.

Without knowing the values of the information symbols I_k beforehand, we model each symbol I_k as a random variable. Assume that the information sequence $\{I_k\}$ is WSS. We choose a linear equalizer $H_E(z)$ to minimize the MSE between the original information symbols I_k and the output of the equalizer $\hat{I}_k$:

$$MSE = E\left[e_k^2\right] = E\left[\left(I_k - \hat{I}_k\right)^2\right] \qquad (3.8)$$

Let us employ the FIR filter of order 2L+1 as the equalizer. We note that a delay of L symbols is incurred at the output of the FIR filter. Then

$$MSE = E\left[\left(I_k - \sum_{j=-L}^{L}\hat{I}_{k-j}h_{E-j}\right)^2\right] = E\left[\left(I_k - \tilde{I}_k^T h_E\right)^2\right] \qquad (3.9)$$

Where

$$\tilde{I}_k = \left[\tilde{I}_{k+L}.......\tilde{I}_{k-L}\right]^T, \qquad (3.10)$$

$$h_E = \left[h_{E,-L}........h_{E,L}\right]^T, \qquad (3.11)$$

We want to minimize MSE by suitable choices of $h_{E,-L}........h_{E,L}$. Differentiating with respect to each $h_{E,j}$ and setting the result to zero, we get

$$E\left[\tilde{I}_k\left(I_k - \tilde{I}_k^T h_E\right)\right] = 0. \qquad (3.12)$$

Rearranging, we get

$$Rh_E = d, \qquad (3.13)$$

Where

$$R = E\left[\tilde{I}_k\tilde{I}_k^T\right] \qquad (3.14)$$

$$d = E\left[I_k\tilde{I}_k^T\right] \qquad (3.15)$$

If R and d are available, then the MMSE equalizer can be found by solving the linear matrix equation(3.13). It can be shown that the signal-to-noise ratio at the output of the MMSE equalizer is better than that of the zero-forcing equalizer.

The linear MMSE equalizer can also be found iteratively. First, notice that the MSE is a quadratic function of h_E. The gradient of the MSE with respect to h_E gives the direction to change h_E for the largest increase of the MSE. In our notation, the gradient is $-2\left(d - Rh_E\right)$. To decrease the MSE, we can update h_E in the direction opposite to the gradient. This is the steepest descent algorithm: At the kth step, the vector $h_E(k)$ is updated as

$$h_E(k) = h_E(k-1) + \mu[d - Rh_E(k-1)] \qquad (3.16)$$

Where μ is a small positive constant that controls the rate of convergence to the optimal solution. In many applications, we do not know R and d in advance. However, the transmitter can transmit a training sequence that is known a priori by the receiver.

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With a training sequence, the receiver can estimate R and d. Alternatively, with a training sequence, we can replace R and d at each step in the steepest descent algorithm by the rough estimates.

IV. SIMULATION AND RESULTS

(A) Simulation Parameters:

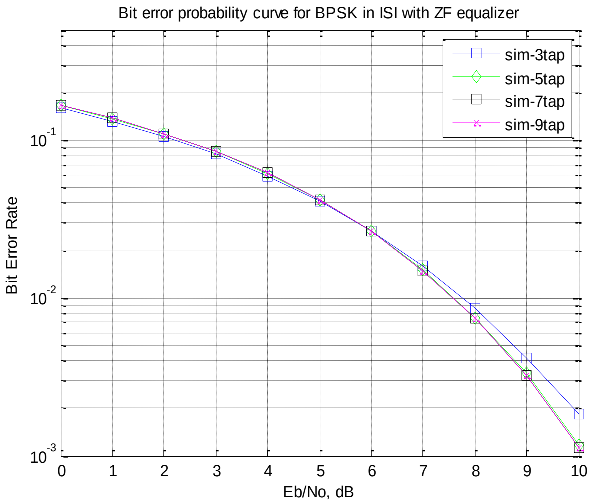
Simulation parameters chosen for the model of OFDM transceiver re listed in Table1.3. Simulation is carried out Rayleigh channel using BPSK modulation technique

Table 1.3 Simulation parameters for OFDM Transceiver

OFDM Parameter	Value
FFT size (nFFT)	64
Number of used subcarriers(nDSC)	52
FFT Sampling frequency	20MHz
Subcarrier spacing	312.5kHz
Used subcarrier index	{-26 to -1, +1 to
Cylic prefix duration (T_{cp})	0.8μs
Data symbol duration (T_d)	3.2μs
Total Symbol duration (T_s)	4μs

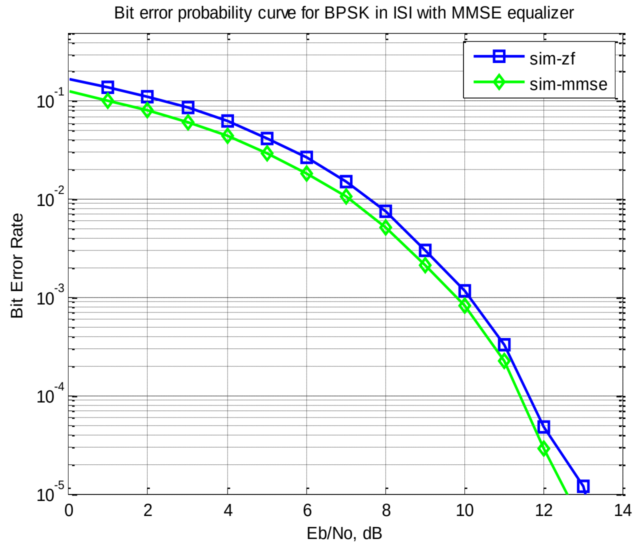
Simulation Results are plotted for bit error rate performance of OFDM System simulation is performed Rayleigh channel using BPSK Modulation technique condition considering absence and presence of Zero forcing and MMSE Equalizer.

In the Graph (4.1) Zero Forcing equalization with 3/5/7/9 tap is performed and the BER computed and the simulation results are as shown in the plot below.



Graph (4.1)

In the Graph (4.2) Minimum Mean Square Error (MMSE) equalization with 7 tap is performed and the BER computed and is compared with Zero Forcing equalization simulation results are as shown in the plot below.



Graph (4.2)

V. CONCLUSION

We compare the BER performance of different equalizers by means of simulation results. Increasing the equalizer tap length from 3 to 5 showed reasonable performance improvement and Diminishing returns from improving the equalizer tap length above 5.

The Zero Forcing Equalizer removes all ISI and is ideal only when the channel is noiseless. When the channel is noisy, the Zero Forcing Equalizer will amplify the noise greatly at frequencies f where the channel response H (j2πf) has a small magnitude (i.e. near zeroes of the channel) in the an attempt to invert the channel completely. The ZF equalizer thus neglects the effect of noise altogether, and is not often used for wireless links. Next step is to discuss the zero forcing equalizer in the presence of transmit pulse shaping and then move on to minimum mean square error equalizer. We can see around 0.5 d_B gain with using MMSE equalizer

From the above following observations we can say that the more balanced linear equalizer is the Minimum Mean Square Error Equalizer, which is does not eliminate ISI completely but instead minimizes the total power of the noise and ISI components in the output. Hence from the above graphs it is evident that the BER decreases as the number of receiving antenna increases with respect to number of transmitting antenna in MMSE equalizer. Comparing ZF and MMSE Equalizer, MMSE is better choice of Equalizer.

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