

# Performance Analysis in Cooperative Communication

Ashima Yadav

**Abstract**— In this paper, we have developed a framework for resource allocation in a multi user cooperative wireless network. We have considered centralized and decentralized node-relay assignment with the objective of maximizing the sum throughput. We present an efficient polynomial time centralized assignment algorithm to achieve this objective, and also investigate efficient decentralized schemes that do not require a central authority to perform the allocation. We present a novel message passing based assignment scheme which converges to the optimal value, and present an adapted scheme based on distributed auction theory. We also present a simple sub-optimal greedy assignment strategy and provide empirical data to support its practical significance.

**Index Terms**—

## I. INTRODUCTION

In the past few decades, wireless communication has been one of the technical areas which has experienced exponential advancement in terms of technology as well as users. This mode of communication alleviates the need of physical connectivity among communicating entities via exploiting the channel as a signal carrying medium. This also makes communication among mobile devices possible. The main objective of any mode of communication is to increase the data rates while also maintaining high reliability (lower probability of error) of the data sent over the communication link. However, the wireless channel suffers from unwanted yet inevitable effects (e.g. shadowing, path loss, multipath fading etc. which make it hard to communicate reliably over the channel. Various diversities of the wireless channels are used as potential solutions to mitigate some of these channel impairments. Spatial diversity is used to combat the deleterious effect of fading via transmitting the signals from different locations, thereby allowing independently faded versions of the signal at the receiver. The idea of multiple input multiple output (MIMO) system was proposed to generate spatial diversity by equipping the wireless device with multiple antennas. However many wireless devices are limited by size and hardware complexity to one antenna and MIMO is not realizable in these cases.

Cooperative communications provides an alternative solution for this problem via enabling single antenna wireless devices in a multi-user environment to share their antennas and generate a virtual multi-antenna transmitter in order to achieve spatial diversity. The broadcast and multi-cast nature of the wireless channel is exploited in cooperative communications. The wireless devices which 'overhear' the transmission between two intended entities can forward the overheard information and provide another independent

faded version of the information at the receiver. Thus, each device in the network is supposed to transmit its own information as well as cooperate in delivering the information originating from other devices.

## II. OBJECTIVE

Our main objective in this project is to study various existing schemes for resource allocation in cooperative communications and to develop novel approaches for the same. There are two fundamental paradigms that fall under the rubric of resource allocation - centralized and decentralized. The criterion for optimality we have selected is maximization of sum throughput. We intend to develop a holistic framework to address both these approaches, with the aim of maximizing the sum throughput in a multi user cooperative wireless network.

## III. SYSTEM MODEL

We have considered a wireless ad-hoc network with  $N_s$  source-destination pairs, and  $N_r$  potential candidates for relaying. Each source-destination pair has its own dedicated channel. Every candidate relay also operates at a different frequency band. If a relay node is cooperating with a given user, it overhears the source information and communicates it to the receiver. Simultaneously, the source also communicates with the destination through its dedicated channel. The destination, by virtue of receiving data from multiple channels, applies maximum ratio combining (MRC) technique. Fig.3 represents the diagrammatic system model. In this scheme –

- $h_{sr}$  is the channel gain between the source and the relay
- $h_{sd}$  is the channel gain between the source and the destination –
- $h_{rd}$  is the channel gain between the relay and the destination

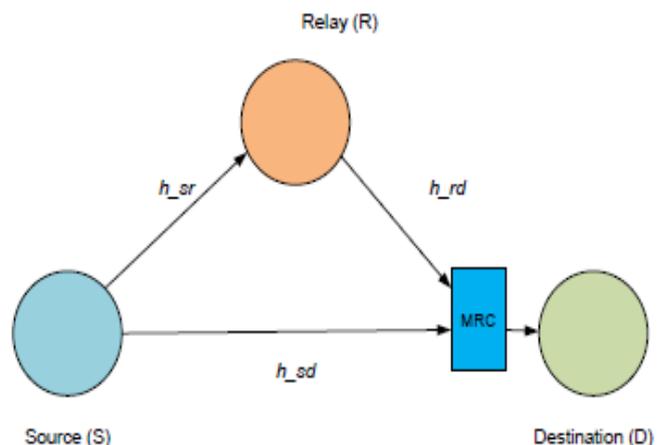


Fig 2 System Model

Manuscript published on 30 October 2012.

\* Correspondence Author (s)

Ashima Yadav\*, Student M.TECH, ECE, Gurgaon Institute of Technology, Bilaspur, Gurgaon, Haryana, India.

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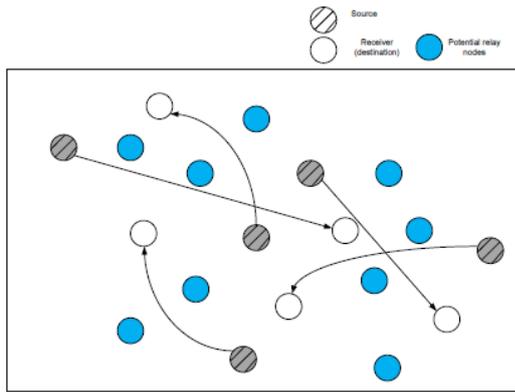


Fig 1 Typical Ad hoc network

## IV. CENTRALIZED SCHEME

There has been much research on selecting an optimal relay node from a set of candidate nodes for a single source-destination pair. In a wireless ad hoc network, there are multiple source-destination pairs competing for the same pool of relays. Our focus is therefore, to develop a criterion to perform an optimal relay assignment for all users in a network from amongst a set of available relays. We will consider assignments that will maximize the sum throughput of the network, while assigning at most one relay to a source-destination pair. A centralized scheme assumes complete information about all participants in the network. In our case, this translates to channel state information and SNR for all participating links, thereby enabling us to compute the requisite link capacities. In [1] Shi et al. have developed an efficient centralized node relay assignment scheme that maximizes the minimum capacity over all possible assignments. We have studied and understood their approach to develop our own model for analysis. We have also simulated the algorithm proposed by them. We shall now proceed to develop a centralized relay assignment algorithm to achieve sum rate maximization in the given network setting.

### 4.1 Problem Statement

We are given  $N_s$  source-destination pairs and  $N_r$  relays. We are given the capacity  $C_{ip}$  for the direct channel between the source and destination, as well as the capacities  $C_{ij}$  between the  $i$ th source-destination communicating through the  $j$ th relay channel, for all  $j$  and  $i$ . Our aim is to assign each source-destination pair to at most one unique relay, such that this assignment maximizes the sum throughput of the network. In other words, there exists no other assignment which can have greater sum throughput.

#### 4.1.1 A Computationally Inefficient Solution

Given all the link throughput values, we are required to come up with the optimal assignment as stated in equation 4.5. A brute force solution to the above stated problem would be to compute the sum rate for all possible assignments, and to select the one which maximizes the required sum metric. As can be easily seen, this problem has a running time that is exponential in the input size. More specifically it will take  $O((N_r + N_s)(N_r + N_s - 1) \dots (N_r))$ , which is computationally intractable. Therefore, there is a need to develop an efficient solution to the above problem which is computationally efficient (polynomial running time).

### 4.2 Solution

We have formulated a graph theoretic solution to the above problem based on networkflows. We model the given problem as a weighted bipartite graph matching problem. Graph matching problems are well studied in algorithms literature. Bipartite Matching has been discussed in [4]. We use the augmenting path approach to generate an optimal assignment. The problem model, along with the algorithm, proof of correctness and complexity analysis are detailed herewith.

#### 4.2.1 Graph Construction

Let  $G$  be a bipartite graph with vertex set  $V = X \cup Y$ , where  $X, Y$  are disjoint sets.

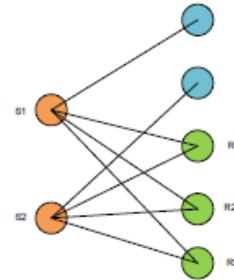


Figure 3: Example Graph Construction

Each vertex in  $X$  represents a unique source-destination pair  $|X| = N_s$

Each vertex in  $Y$  represents a unique relay or a direct channel between some source-destination pair.  $|Y| = N_r + N_s$ .

We construct the edge set  $E$  as follows -

For every node  $i \in X$  there exists an edge  $e_{ij}$  between  $i \in X$  and  $j \in Y$ , if it is possible for the said source-destination pair to use the  $j$ th channel. Each such edge  $e_{ij}$  will have an associated non-negative real number  $c_{ij}$ .

**Note:** The  $N_s$  direct channels represented by nodes in  $Y$  will have one edge each corresponding to their source-destination. The relays will have edges corresponding to multiple nodes in  $X$ .

The Fig.5 gives an example construction of such a bipartite graph for  $N_s = 2, N_r = 3$ .

#### 4.2.2 Definitions

##### 4.2.2.1 Bipartite Graph

A bipartite graph  $G$  can be partitioned into two disjoint vertex sets  $V = X \cup Y$ , such that every edge  $e \in E$  has one end in  $X$  and the other in  $Y$ .

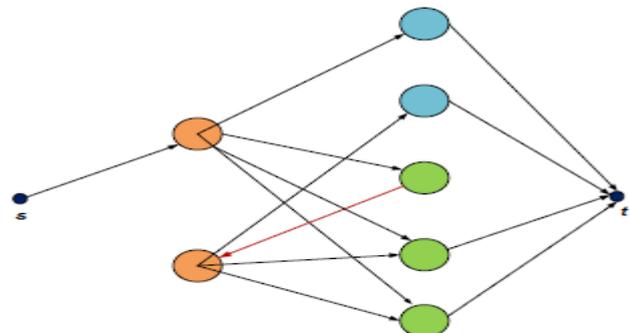


Figure 4: Residual Graph for Matching,  $|M| = 1$



#### 4.2.2.2 Matching

A matching  $M$  in  $G$  is a subset of edges  $M \subseteq E$  such that each vertex appears in at most one edge in  $M$ .

- The size of a matching is  $|M|$
- The cost of a matching is  $C_M = \sum_{e_{ij} \in M} c_{ij}$

In our construction,  $M$ , such that  $|M| = N_s$  is the minimum cost perfect matching if

$$C_M \leq C_{M'}; \forall M' : |M'| = N_s$$

#### 4.2.2.3 Residual Graph

Let  $M$  be a matching. We add two new nodes  $s$  and  $t$  to the graph. We add edges  $(s, x)$  for all nodes  $x \in X$  that are unmatched and edges  $(y, t)$  for all nodes  $y \in Y$  that are unmatched. An edge  $e = (x, y)$  is directed from  $x$  to  $y$  if  $e \notin M$ , and from  $y$  to  $x$  if  $e \in M$ . For all edges oriented from  $y$  to  $x$ , we replace the edge weight  $c_{xy}$  with the negative value  $-c_{xy}$ . All edges of the form  $(s, x)$  and  $(y, t)$  are assigned weight 0. The weighted, directed, bipartite graph so obtained for a given matching  $M$  is referred to as the residual graph  $GM$ .

#### 4.2.2.4 Augmenting Path

In the residual graph  $GM$ , let  $P$  be a directed path from  $s$  to  $t$ . If such a path exists, then we can construct a matching  $M'$ , s.t.  $|M'| = |M| + 1$ . This can be achieved in the following manner-

- if edge  $e$  in path  $P$  is directed from  $X$  to  $Y$ , then it is added to  $M$
- if edge  $e$  in path  $P$  is directed from  $Y$  to  $X$ , then it is deleted from  $M$

We call  $P$ , the augmenting path and use it to augment the matching  $M$

#### 4.2.3 Algorithm

We now present the optimal node relay assignment algorithm to maximize sum capacity.

##### Main Algorithm

- 1:  $N_s$  = number of source-destination pairs
- 2:  $N_r$  = number of relay channels
- 3: Construct Graph  $G$
- 4:  $S = \max(\text{capacity}(I, E_j))$  over all possible  $e(i, j) \in E$
- 5: for every edge  $e_{ij} \in E$ ,  $c_{ij} = S \times \text{capacity}(i, j)$
- 6:  $m = 0$
- 7:  $M = \emptyset$
- 8: while  $m \neq N_s$  do
- 9: construct  $GM$ , the residual graph for matching  $M$
- 10: compute the shortest path  $P$  from  $s$  to  $t$  in  $GM$
- 11: call Augment( $P, M, GM$ )
- 12:  $m = m + 1$
- 13: end while
- 14:  $M$  is the minimum cost perfect matching of size  $N_s$
- 15:  $e(x, y) \in M$  assign  $x$ th source destination pair to  $y$ th relay channel

Augment(Path  $P$ , Matching  $M$ , Graph  $GM$ )

- 1: for every edge  $e \in P$  do
- 2: if  $e$  is directed from  $X$  to  $Y$  in  $GM$  then

- 3:  $e = (x, y)$
- 4: Add  $e$  to  $M$
- 5: else if  $e$  is directed from  $Y$  to  $X$  in  $GM$  then
- 6:  $e = (y, x)$
- 7: Delete  $(x, y)$  from  $M$
- 8: else continue to next edge in  $P$
- 9: end if
- 10: end if

#### 4.2.4 Proof of Correctness

In the above algorithm, we are iteratively constructing a matching of size  $k + 1$  from a matching of size  $k$ . We claim that the final matching of size  $N_s$  obtained, is the minimum cost matching.

Observation: Given a matching  $M$  and a path  $P$  from  $s$  to  $t$  in  $GM$ , let  $M_0$  be the matching obtained by augmenting path  $P$  to  $M$ . Then  $|M_0| = |M| + 1$  and  $C_{M_0} = C_M + \text{cost}(P)$ . Our aim therefore is to iteratively augment along paths of minimum cost to obtain a matching of size  $N_s$ .

A Negative Cycle is a collection of vertices forming a closed loop path such that the sum of edge weights  $< 0$ .

Lemma: Let  $M$  be a matching of size  $N_s$ . If there exists a negative cost cycle  $C$  in  $GM$ , then  $M$  is not minimum cost.

Proof: Note that  $s$  and  $t$  cannot be part of a cycle in  $GM$ , because there are no edges emerging from  $s$  or  $t$ . If we were to use the edges of cycle  $C$  to augment the matching  $M$ , we will get another matching  $M_0$  of size  $N_s$ . However,  $C_{M_0} = C_M + \text{cost}(C)$  where  $\text{cost}(C) < 0$ . Thus,  $M$  is not of minimum cost.

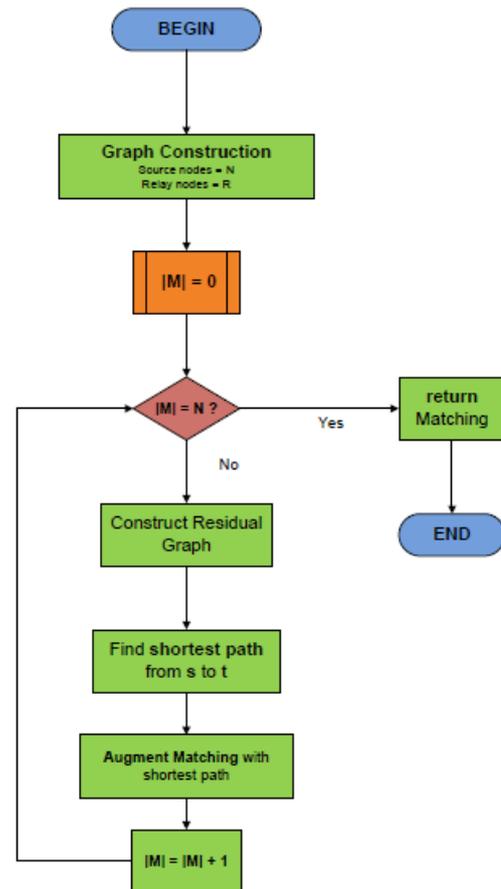


Figure 5: Description of Algorithm

## Performance Analysis in Cooperative Communication

Lemma: Let  $M$  be a matching of size  $N_s$ . If there are no negative cost directedcycles in  $GM$ , then  $M$  is a minimum cost perfect matching.

Proof: Let  $M'$  be a matching of size  $N_s$  of smaller cost than  $M$ . Consider the set of edges  $E = M \oplus M'$ . This edge set corresponds to a set of vertex disjoint directedcycles in  $GM$ . The cost of this set is  $CM' - CM < 0$ . Thus, it must be that at least one of the cycles has negative cost.

With the above result, it is clear that we want to generate matchings of larger and larger sizes while ensuring that in no iteration do we have a negative cycle. This is the motivation behind choosing the smallest weight path from  $s$  to  $t$  as our augmenting path. When we terminate with a matching of size  $N_s$  in the above algorithm, it is the one that has minimum cost) Maximum Sum throughput.

Example 5.1: We presented the algorithm with the throughput values given in Table 5.1 (for a multiuser network with  $N_s = 5$  and  $N_r = 4$ ).

For the above data, our algorithm presented us with the optimal assignment to maximize sum throughput. The returned assignment has been indicated in the table itself.

Source-Destination	R1	R2	R3	R4	$\phi$
1	20	25	28	<u>30</u>	8
2	43	<u>63</u>	45	31	29
3	39	41	34	35	<u>14</u>
4	37	41	<u>56</u>	44	33
5	<u>59</u>	60	39	37	23

Table 1. 1: Throughput matrix of Example 3

The total sum throughput in the optimal assignment = 222

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**Ashima Yadav** is a Student of M Tech Final Year in Electronics and Communication at Gurgaon Institute of Technology and Management, Bilaspur, Gurgaon, Haryana. Affiliated to M.D. University, Rohtak, Haryana. She has done her B.Tech in Electronics and Communication from GLA University, UP, India. She has Also Done her MBA in Telecom Management from Annamalai University, India.