International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249 – 8958, Volume-1 Issue-5, June 2012

Effect of Shape Factor on the Flexural-Torsional-Distortional Behaviour of Thin- Walled Box Girder Structures

C.A. Chidolue, C.H. Aginam

Abstract: ***The governing differential equations of equilibrium for flexural-torsional-distortional analysis of thin-walled box girder structures with various shapes were derived in this work using V. Z. Vlasov's theory. The obtained equations were used to evaluate the cross sectional deformations of some box girder structures having rectangular (doubly symmetric) and trapezoidal (mono symmetric) shapes. Evaluation of Vlasov's coefficients for the obtained differential equations of equilibrium formed a major part of this work. This was accomplished by examining the strain modes interaction diagrams for the various cross sections and by using Morh's integral chart for displacement computations. Cross sectional deformations of the box girder structures were obtained by integration of the differential equations of equilibrium using method of trigonometric series with accelerated convergence. For irregular (asymmetric) box girder shapes, complex differential equations of equilibrium were obtained as a result of the interaction between all the strain modes of flexure, torsion and distortion. Application of these set of equations for the analysis of irregular shaped box girder structures is presented in another work.***

Keywords: Box structure, distortion, flexure, shape factor, thin-walled, torsion

I. INTRODUCTION

When a thin-walled box girder is restrained from warping, additional stresses arise in the longitudinal and transverse directions. These stresses do not arise in the case of uniform (Saint Venant) torsion.

Every cross section has a shear centre which is the point through which the shear force must be applied if there is to be no twisting. When a cross section has two planes of symmetry (doubly symmetric), the shear centre S and the centre of gravity G coincide. If there is one plane of symmetry (e.g. as there is for a channel section), S and G are located on the same plane and it is a relatively simple matter to establish their coordinates.

However when the cross section is less regular a systematic method for locating the shear centre is required and the development of equations of equilibrium for flexural-torsional-distortional analysis of box girders becomes more complex.

Manuscript Received on June 2012.

C.A. Chidolue, Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria

C.H. Aginam, Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria.

For the context of this work, shape factor refers to different box girder cross sections distinguished by their shapes having, (a) two planes of symmetry (b) one plane of symmetry, and (c) no plane of symmetry.

The theory of thin-walled structures whose transverse cross sections are arbitrary and which contains several closed contours was developed by V.Z.Vlasov [1].

Vlasov's coefficients of differential equations of equilibrium involve a combination of elementary displacements. In the general strain mode there are three strain fields in the longitudinal direction and four in the transverse direction. These are;

Out of plane displacement due to vertical load characterized by φ_1.

Out of plane displacement due to horizontal load normal to box girder axis, characterized by φ_2.

Out of plane displacement due to warping of the cross section, characterized by φ_3.

In-plane displacement due to vertical load, characterized by ψ_1.

In-plane displacement due to horizontal load normal to box girder axis, characterized by ψ_2.

In-plane displacement due to distortion of the cross section, characterized by ψ_3.

In-plane displacement due to pure rotation, characterized by ψ_4.

Some or all of these strain modes may be present in a given frame depending on the shape of the cross section and the nature of loading.

Fig. 1 shows that torsional loads consisting of opposing vertical forces result from gravity loads that are eccentric to the centre line of the girder and they give rise to bending and torsion. Fig. 2 shows that a torsional load can be modeled as a uniform (Saint Venant) torsion and a distortional component. Therefore an eccentric load on a box girder structure introduces interaction between bending, pure torsion and distortion. The flexural- torsional-distortional interaction is of great interest particularly in a thin walled box structure where the shape of the cross section comes into play.

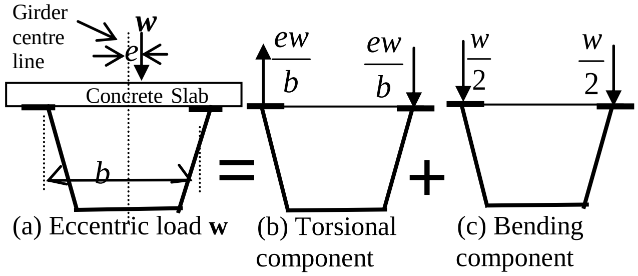


Fig. 1 Decomposition of eccentric concrete load on trapezoidal box girder

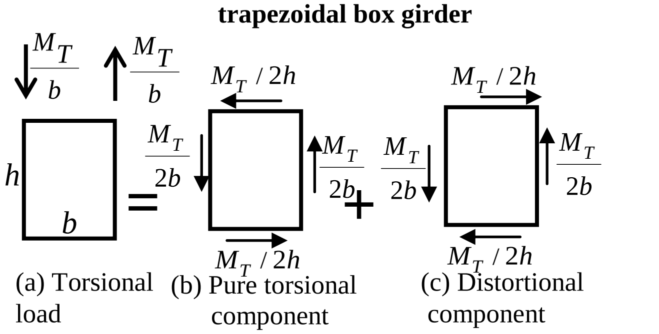


Fig.2 Torsional and distortional loads on rectangular box girder due to vertical forces

Research [2], has shown that a doubly symmetric section has only one interaction of strain modes, i.e. torsional strain mode and distortional strain mode interaction. A mono-symmetric section on the other hand, has three strain modes interactions: torsion interacts with distortion and each of these interacts with flexure about the non axis of symmetry. Thus we have torsional-distortional interaction, flexural-torsional interaction, and flexural-distortional interaction.

A non symmetric section has multiple strain modes interactions: each of torsion and distortion interacts with flexure about both axes of non-symmetry in addition to the interaction between themselves. Thus, we have torsiona-distortional interaction, torsional-flexural interaction in major axis, torsional-flexural interaction in minor axes, flexural-distortional interaction in major axis and flexura-distortional interaction in minor axis. These strain modes interactions are quite inseparable that it is not possible to examine one strain modes interaction in isolation of others.

In this work, the flexural-torsional-distortional interactions in doubly symmetric (rectangular) profile, mono symmetric (trapezoidal) profile and non symmetric (irregular) profile, were investigated. The governing differential equations of equilibrium were derived for flexural-torsional-distortional analysis of the box girder frames. By using a single cell doubly symmetric section and a single cell mono symmetric section the derived equations were used to obtain the cross sectional deformations of the box girder structures. The application of the derived equations in the analysis of non symmetric box girder structures form the basis for another work.

II. REVIEW OF PAST WORK

The curvilinear nature of box girder bridges along with their complex deformation patterns and stress fields have led designers to adopt approximate and conservative methods for their analysis and design. Recent literatures: Hsu et al [3], Fan and Helwig [4], Sennah and Kennedy [5], on straight and curved box girder bridges deal with analytical formulations to better understand the behaviour of these complex structural systems. Few authors Okil and El-tawil [6], Sennah and Kennedy [5], have undertaken experimental studies to investigate the accuracy of existing methods. Before the advent of Vlasov's 'theory of thin-walled beams' [1], the conventional method of predicting warping and distortional stresses is by beam on elastic foundation (BEF) analogy. This analogy ignores the effect of shear deformations and takes no account of the cross sectional deformations which are likely to occur in a thin walled box girder structure

Several investigators Paavola [7], Razaqpur and Lui [8], Fu and Hsu [9], Tesar [10], have combined thin-walled beam theory of Vlasov and the finite element technique to develop a thin walled box element for elastic analysis of straight and curved cellular bridges. Osadebe and Chidolue [11], [12], Eze [13], obtained fourth order differential equations of torsional-distortional equilibrium for the analysis of mono symmetric box girder structures using Vlasov's theory with modifications by Varbanov [14].

Various theories were therefore postulated by different authors examining methods of analysis, both classical and numerical. A few others however carried out tests on prototype models to verify the authenticity of the theories. At the end of it all, we concluded that Vlasov's theory captures all peculiarities of cross sectional deformations such as warping, torsion, distortion etc, and is therefore adopted in this work.

III. ENERGY FORMULATION OF THE EQUILIBRIUM EQUATIONS

The potential energy of a box structure under the action of a distortional load of intensity q is given by [11]:

$$\Pi = \frac{E}{2}\sum a_{ij}U_i{}'(x)U_j{}'(x)dx + \frac{G}{2}\Big[\sum b_{ij}U_i(x)U_j(x) + \sum c_{kj}U_k(x)V_j{}'(x)\Big]dx + \frac{G}{2}\Big[\sum c_{ih}U_i(x)V_h{}'(x) + \sum r_{kh}V_k{}'(x)V_h{}'(x)\Big]dx + \frac{E}{2}\sum s_{hk}V_k(x)V_h(x)dx - \sum q_h V_h dx \qquad (1)$$

where, U_i, U_j are unknown functions governing the variation of out of plane displacements along the length of the box girder frame.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$V_h(x), V_k(x)$ are unknown functions governing the variation of in plane displacements along the length of the girder frame. The coefficients a_{ij}, b_{ij}, c_{kj}, c_{ih}, s_{hk} are called Vlasov's coefficients and are given as follows:

$$a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA \qquad \text{(a)}$$
$$b_{ij} = b_{ji} = \int \varphi_i^{'}(s)\varphi_j^{'}(s)dA \qquad \text{(b)}$$
$$c_{kj} = c_{jk} = \int \varphi_k^{'}(s)\psi_j(s)dA \qquad \text{(c)}$$
$$c_{ih} = c_{hi} = \int \varphi_i^{'}(s)\psi_k(s)dA \qquad \text{(d)} \qquad (2)$$
$$r_{kh} = r_{hk} = \int \psi_k(s)\psi_h(s)dA; \qquad \text{(e)}$$
$$s_{kh} = s_{hk} = \frac{1}{E}\int \frac{M_k(s)M_h(s)}{EI_{(s)}}ds \qquad \text{(f)}$$
$$q_h = \int q\psi_h ds \qquad \text{(g)}$$

The governing equations of torsional-distortional equilibrium are obtained by minimizing the energy functional (1), with respect to its functional variables U(x) and V(x) using Euler Lagrange technique, [15]. Minimizing with respect to U(x) we obtain;

$$k\sum_{i=1}^{m} a_{ij}U_i\,''(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k\,'(x) = 0 \qquad (3)$$

Minimizing with respect to V(x) we have;

$$\sum c_{ih}U_i\,'(x) - \kappa\sum s_{hk}V_k(x) + \sum r_{kh}V_k\,''(x) + \frac{1}{G}\sum q_h = 0 \qquad (4)$$

where $\kappa = E/G = 2(1+\nu)$

Equations (3) and (4) are Vlasov's generalized differential equations of distortional equilibrium for a box girder. They are presented in matrix form as follows:

$$\kappa\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{Bmatrix} U_1^{''} \\ U_2^{''} \\ U_3^{''} \end{Bmatrix} - \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix}$$
$$-\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{bmatrix}\begin{Bmatrix} V_1^{'} \\ V_2^{'} \\ V_3^{'} \\ V_4^{'} \end{Bmatrix} = 0 \qquad \text{(5a)}$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{bmatrix}\begin{Bmatrix} U_1^{'} \\ U_2^{'} \\ U_3^{'} \end{Bmatrix} - \kappa\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{41} & s_{43} & s_{44} \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix}$$
$$+\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix}\begin{Bmatrix} V_1^{''} \\ V_2^{''} \\ V_3^{''} \\ V_4^{''} \end{Bmatrix} + \frac{1}{G}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = 0 \qquad \text{(5b)}$$

IV. STRAIN MODES INTERACTION AND DIAGRAMS

A close examination of Vlasov's coefficients of the differential equations of equilibrium given in (2a) to (2f) show that the coefficients $a_{ij} = a_{ji}$ represent the interaction between all out of plane displacements characterized by φ_1, φ_2 and φ_3. If either φ_i or φ_j is zero the value of a_{ij} is zero, indicating that there was no interaction between the two strain modes. Similarly, $b_{ij} = b_{ji}$ represent the interaction between all in plane displacements of the box girder frame characterized by $\varphi_1\,'$, $\varphi_2\,'$ and $\varphi_3\,'$, where the primes $'$ indicate first derivative of the functions. Again, if either $\varphi_i\,'$ or $\varphi_j\,'$ is zero the value of b_{ij} is zero, indicating that there was no interaction between the two strain modes.

The coefficients $c_{kj} = c_{jk}$ or $c_{ih} = c_{hi}$ represent the interaction between (the derivatives of) all out of plane displacement strain modes $\varphi_1\,'$, $\varphi_2\,'$ and $\varphi_3\,'$ and all in plane displacement strain modes, ψ_1, ψ_2, ψ_3 and ψ_4. The coefficients $r_{kh} = r_{hk}$ represent the interaction between all in plane displacement strain modes ψ_1, ψ_2, ψ_3 and ψ_4 while $s_{hk} = s_{kh}$ represent the interaction between the distortional bending moments arising from all the strain modes. Incidentally, only φ_3 strain mode give rise to distortional bending moment as all other strain modes do not amount to distortion of the cross section. Hence:

$$s_{hk} = s_{kh} = s_{33} = \frac{1}{E}\int \frac{M_3(s)M_3(s)}{EI_{(s)}}ds \qquad (6)$$

Figs. 3, and 5, show the strain modes diagrams for rectangular cross section and irregular cross section respectively, used for numerical study in this work. Fig. 4 shows a single cell trapezoidal box girder section also used for numerical analysis. The strain modes diagrams for this box girder cross section is given in [11]. The coefficients $a_{ij}, b_{ij}, c_{kj}, c_{ih}$ and r_{kh}, of the governing equations of equilibrium are computed with the aid of Morh's integral chart using the strain modes diagrams. The procedure for obtaining strain modes diagrams is available in literatures [11], [17]. The summary of the coefficients is given on Table 1.

V. FORMULATION OFDIFFERENTIAL EQUATIONS OF EQUILIBRIUM FOR VARIOUS BOX GIRDER SECTIONS

General expressions for the differential equations of equilibrium for box girders are given by (3) and (4). In this section, specific expressions for flexural-torsional-distortional equilibrium equations are obtained for rectangular (doubly symmetric) section, trapezoidal (mono-symmetric) section and irregular (non symmetric) box girder structures.

A. Rectangular (doubly symmetric) section

The relevant coefficients for flexural-torsional-distortional equilibrium are those involving strain modes 2, 3 and 4. These are $a_{22},\ a_{23},\ a_{33},\ b_{22}, b_{23},\ c_{22},\ c_{23},\ r_{22}, b_{33}, c_{33},$ $c_{24} = c_{42},\ c_{34} = c_{43},\ r_{34} = r_{43},\ r_{24}, r_{44},$ and s_{33}. Because of symmetry we note that there is no interaction between strain modes 2 and 3 and between strain modes 2 and 4. Consequently, $a_{23}, b_{23}, c_{23}, r_{23}, c_{24}, c_{42}, r_{24}, r_{42}$ are zero as can be seen fron Table 1. Substituting these coefficients into (5a) and (5b) we obtain:

$$k\begin{bmatrix}0&0&0\\0&a_{22}&0\\0&0&a_{33}\end{bmatrix}\begin{pmatrix}U_1''\\U_2''\\U_3''\end{pmatrix}-\begin{bmatrix}0&0&0\\0&b_{22}&0\\0&0&b_{33}\end{bmatrix}\begin{pmatrix}U_1\\U_2\\U_3\end{pmatrix}$$

$$-\begin{bmatrix}0&0&0&0\\0&c_{22}&0&0\\0&0&c_{33}&c_{34}\end{bmatrix}\begin{pmatrix}V_1'\\V_2'\\V_3'\\V_4'\end{pmatrix}=0 \qquad (7)$$

$$\begin{bmatrix}0&0&0\\0&c_{22}&0\\0&0&c_{33}\\0&0&c_{43}\end{bmatrix}\begin{bmatrix}U_1'\\U_2'\\U_3'\end{bmatrix}-k\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&s_{33}&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}V_1\\V_2\\V_3\\V_4\end{bmatrix}$$

$$+\begin{bmatrix}0&0&0&0\\0&r_{22}&0&0\\0&0&r_{33}&r_{34}\\0&0&r_{43}&r_{44}\end{bmatrix}\begin{bmatrix}V_1''\\V_2''\\V_3''\\V_4''\end{bmatrix}+\frac{1}{G}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}=0 \qquad (8)$$

Expanding (7) and (8) we obtain the following equations.

$$ka_{22}U_2'' - b_{22}U_2 - c_{22}V_2' = 0 \qquad (9)$$

$$ka_{33}U_3'' - b_{33}U_3 - c_{33}V_3' - c_{34}V_4' = 0 \qquad (10)$$

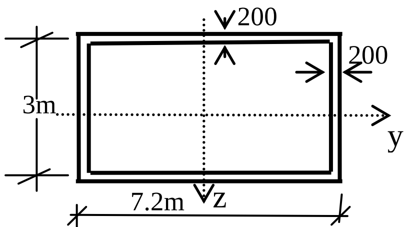
$$c_{22}U_2' + r_{22}V_2'' + q_2/G = 0 \qquad (11)$$

$$c_{33}U_3' - ks_{33}V_3 + r_{33}V_3'' + r_{34}V_4'' = -q_3/G \qquad (12)$$

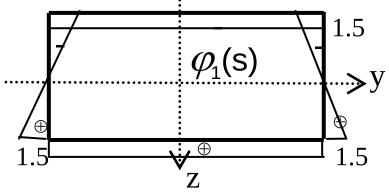
$$c_{43}U_3' + r_{43}V_3'' + r_{44}V_4'' = -q_4/G \qquad (13)$$

Simplifying further by eliminating U_2 and U_3 we obtain:

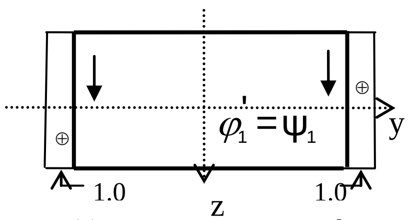
$$\beta_1 V_4'' - \gamma_1 V_3 + K_1 = 0$$
$$V_3^{iv} + \alpha_1 V_4^{iv} + \beta_2 V_4'' = K_2 \qquad (14)$$



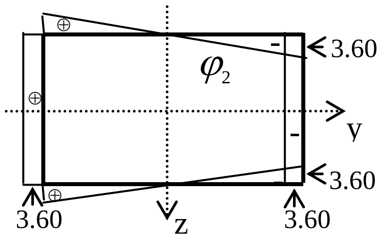
(a) Single cell doubly-svmmetric section



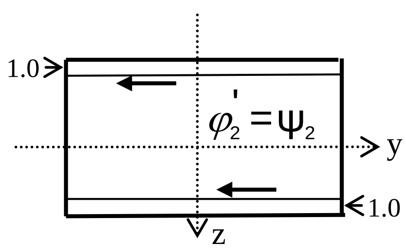
(b) Longitudinal strain mode diagram (bending about y-y axis)



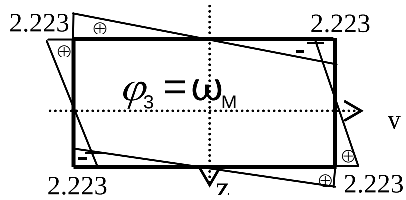
(c) Transverse strain mode in y-direction



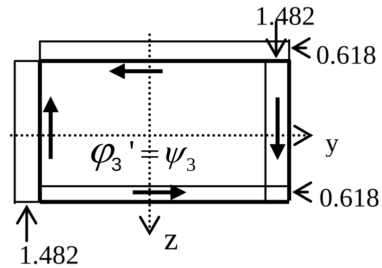
(d) Longitudinal strain mode diagram (bending about z-z axis)



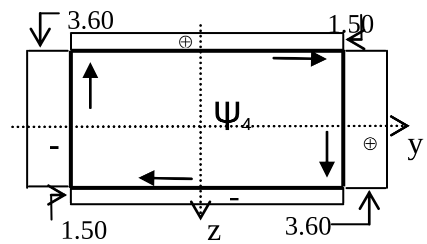
(e) Transverse strain mode in z-direction



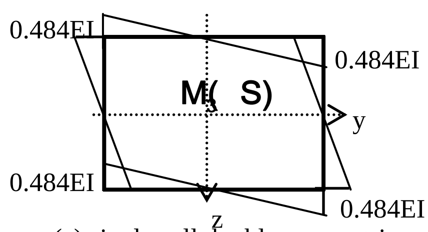
(f) Warping Function



(g) Distortion diagram



(h) Rotation diagram



(a) single cell doubly symmetric section

Fig. 3 Generalized strain modes for single cell rectangular box girder section

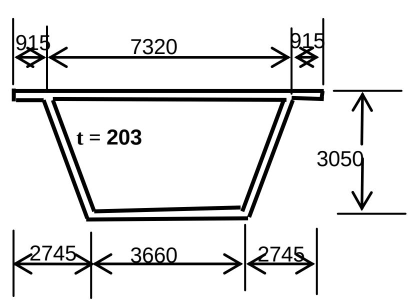
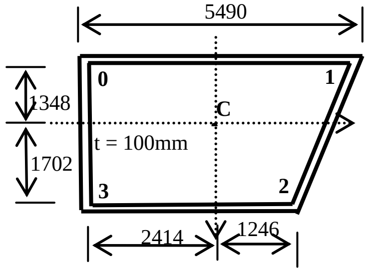
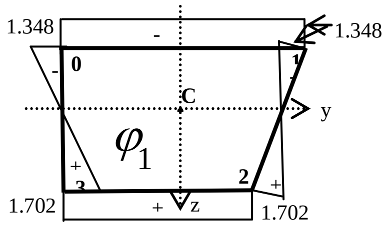


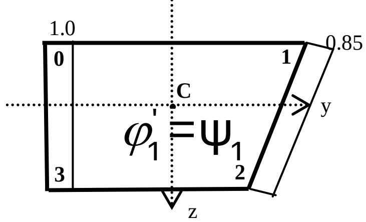
Fig. 4 Single cell trapezoidal box girder section



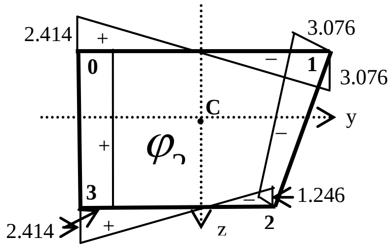
(a) Single cell non symmetric cross section



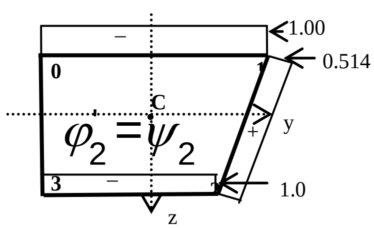
(b) Longitudinal strain mode: bending in y-y direction



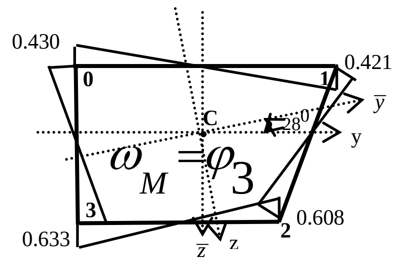
(c) Transverse strain mode in y-y direction



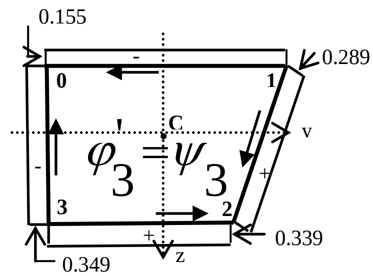
(d) Longitudinal strain mode: bending in z-z direction



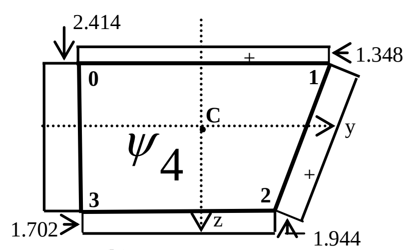
(e) Transverse strain mode in z-z direction



(f) Warping function



(g) Distortion Diagram



(h) Rotation

Fig.5 Generalized strain modes for single cell mono-symmetric box girder frame

Table I: Summary of Vlasov's coefficients

Coefficient elements	Single cell rectangular section	Single cell trapezoidal section	Single cell irregular section
a_{11}	10.930	6.453	5.264
$a_{12} = a_{21}$	0.00	0.00	1.536
$a_{13} = a_{31}$	0.00	0.00	-0.001
a_{22}	27.994	25.050	11.018
$a_{23} = a_{32}$	0.00	-0.270	-0.038
a_{33}	6.721	0.757	0.293
b_{11}	1.200	1.060	1.132
$b_{12} = b_{21}$	0.00	0.00	0.314
$b_{13} = b_{31}$	0.00	0.00	-0.037
b_{22}	2.880	2.982	2.018
$b_{23} = b_{32}$	0.00	-1.066	0.028
b_{33}	3.736	1.407	0.244
c_{11}	1.200	1.060	1.132
$c_{12} = c_{21}$	0.00	0.00	0.314
$c_{13} = c_{31}$	0.00	0.00	-0.037
$c_{14} = c_{41}$	0.00	0.00	0.148
c_{22}	2.880	2.982	2.018
$c_{23} = c_{32}$	0.00	-1.066	0.028
c_{24}	0.00	-1.561	0.476
c_{33}	3.736	1.407	0.244
c_{34}	3.732	1.265	0.262
r_{11}	1.200	1.060	1.132
$r_{12} = r_{21}$	0.00	0.00	0.314
$r_{13} = r_{31}$	0.00	0.00	-0.037
$r_{14} = r_{41}$	0.00	0.00	0.148
r_{22}	2.880	2.982	2.018
$r_{23} = r_{32}$	0.00	-1.066	0.028
$r_{24} = r_{42}$	0.00	-1.561	0.476
r_{33}	3.736	1.407	0.244
$r_{34} = r_{43}$	3.732	1.265	0.262
r_{44}	22.032	14.616	10.358
s_{33}	$4.338*10^{-2}$	$1.820*10^{-4}$	-

where

$$\alpha_1 = r_{44}/c_{43};$$

$$\beta_1 = (r_{34}/c_{33} - r_{44}/c_{43})$$

$$\beta_2 = (c_{34}c_{43} - b_{33}r_{44})/ka_{33}c_{43} \qquad \gamma_1 = ks_{33}/c_{33}$$

$$K_1 = (q_3/c_{33}G) - (q_4/c_{43}G)$$

$$K_2 = b_{33}q_4/a_{33}c_{43}E$$

B. Trapezoidal (mono- symmetric) Section

The relevant coefficients for flexural-torsional-distortional equilibrium are those involving strain modes 2, 3 and 4. These are; a_{32}, a_{33}, b_{32}, b_{33}, c_{32}, c_{33}, c_{34}, c_{42}, r_{22}, r_{32}, r_{33}, r_{34}, r_{42}, s_{33}, and r_{44}. Substituting other coefficients as zero in (5a) and (5b), noting that $a_{ij} = a_{ji}$, $b_{ij} = b_{ji}$, etc, we obtain:

$$\kappa\begin{bmatrix}0 & 0 & 0\\ 0 & a_{22} & a_{23}\\ 0 & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}U_1''\\ U_2''\\ U_3''\end{bmatrix} - \begin{bmatrix}0 & 0 & 0\\ 0 & b_{22} & b_{23}\\ 0 & b_{32} & b_{33}\end{bmatrix}\begin{bmatrix}U_1\\ U_2\\ U_3\end{bmatrix} - \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & c_{22} & c_{23} & c_{24}\\ 0 & c_{32} & c_{33} & c_{34}\end{bmatrix}\begin{bmatrix}V_1'\\ V_2'\\ V_3'\\ V_4'\end{bmatrix} = 0 \qquad (15)$$

$$\begin{bmatrix}0 & 0 & 0\\ 0 & c_{22} & c_{23}\\ 0 & c_{32} & c_{33}\\ 0 & c_{42} & c_{43}\end{bmatrix}\begin{bmatrix}U_1'\\ U_2'\\ U_3'\end{bmatrix} - -\kappa\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & s_{33} & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}V_1\\ V_2\\ V_3\\ V_4\end{Bmatrix}$$

$$+\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&r_{33}&r_{34}\\0&0&r_{43}&r_{44}\end{bmatrix}\begin{Bmatrix}V_1''\\V_2''\\V_3''\\V_4''\end{Bmatrix}+\frac{1}{G}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}=0 \quad (16)$$

Multiplying out we obtain

$$ka_{22}U_2''+ka_{23}U_3''-b_{22}U_2-b_{23}U_3-c_{22}V_2'-c_{23}V_3'$$
$$-c_{24}V_4'=0 \quad (17)$$
$$ka_{32}U_2''+ka_{33}U_3''-b_{32}U_2-b_{33}U_3-c_{32}V_2'-c_{33}V_3'$$
$$-c_{34}V_4'=0 \quad (18)$$
$$c_{22}U_2'+c_{23}U_3'+r_{22}V_2''+r_{23}V_3''+r_{24}V_4''=-q_2/G \quad (19)$$
$$c_{32}U_2'+c_{33}U_3'-ks_{33}V_3+r_{32}V_2''+r_{33}V_3''+r_{34}V_4''$$
$$=-q_3/G \quad (20)$$
$$c_{42}U_2'+c_{43}U_3'+r_{42}V_2''+r_{43}V_3''+r_{44}V_4''=-q_4/G \quad (21)$$

Simplifying further by eliminating U_2 and U_3 we obtain:

$$\beta_4V_2''+\beta_5V_3''+\beta_6V_4''-\gamma_1V_3=-K_3 \quad (22)$$
$$\alpha_4V_2^{iv}+\alpha_5V_3^{iv}+\alpha_6V_4^{iv}-\beta_7V_2''-\beta_8V_3''-\beta_9V_4''$$
$$+\gamma_2V_3=K_4 \quad (23)$$
$$\alpha_7V_2^{iv}+\alpha_8V_3^{iv}+\alpha_9V_4^{iv}-\beta_{10}V_2''-\beta_{11}V_3''-\beta_{12}V_4''$$
$$+\gamma_3V_3=K_5 \quad (24)$$

where,

$$\alpha_1=\left(r_{32}/c_{33}-r_{22}/c_{23}\right)/\left(c_{22}/c_{23}-c_{32}/c_{33}\right)$$
$$\alpha_2=\left(r_{34}/c_{33}-r_{24}/c_{23}\right)/\left(c_{22}/c_{23}-c_{32}/c_{33}\right)$$
$$\alpha_3=Ks_{33}/c_{33}\left(c_{22}/c_{23}-c_{32}/c_{33}\right)$$
$$\alpha_4=ka_{22}\alpha_1;\quad \alpha_5=ka_{23}\beta_1;$$
$$\alpha_6=k(a_{22}\alpha_2+a_{23}\beta_3)\ \alpha_7=ka_{32}\alpha_1;\quad \alpha_8=ka_{33}\beta_1;$$
$$\alpha_9=k(a_{32}\alpha_2+a_{33}\beta_3)$$
$$\beta_1=\left(r_{33}/c_{32}-r_{23}/c_{22}\right)/\left(c_{23}/c_{22}-c_{33}/c_{32}\right)$$
$$\beta_2=\left(r_{34}/c_{32}-r_{24}/c_{22}\right)/\left(c_{23}/c_{22}-c_{33}/c_{32}\right)$$
$$\beta_3=ks_{33}/c_{32}\left(c_{23}/c_{22}-c_{33}/c_{32}\right)$$
$$\beta_4=\left(c_{42}\alpha_1+r_{42}\right);\ \beta_5=\left(c_{43}\beta_1+r_{43}\right);$$
$$\beta_6=\left(c_{42}\alpha_2+c_{43}\beta_2+r_{44}\right);\qquad \beta_7=(b_{22}\alpha_1+c_{22})$$
$$\beta_8=(ka_{23}\beta_3+b_{23}\beta_1+k\alpha_{22}\alpha_3+c_{23});$$
$$\beta_9=-(b_{22}\alpha_2+c_{24}+b_{23}\beta_2);\ \beta_{10}=(b_{32}\alpha_1+c_{32})$$
$$\beta_{11}=(ka_{32}\alpha_3+b_{33}\beta_1+k\alpha_{33}\beta_3+c_{33});$$
$$\beta_{12}=(b_{32}\alpha_2+c_{34}+b_{33}\beta_2)$$
$$\gamma_1=\left(c_{42}\alpha_3+c_{43}\beta_3\right);\ \gamma_2=(b_{22}\alpha_3+b_{23}\beta_3);$$
$$\gamma_3=(b_{32}\alpha_3+b_{33}\beta_3);$$
$$K_1=\bar{q}_3/c_{32}G\left(c_{23}/c_{22}-c_{33}/c_{32}\right)-$$
$$-\bar{q}_2/c_{22}G\left(c_{23}/c_{22}-c_{33}/c_{32}\right)$$
$$K_2=\bar{q}_3/c_{33}G\left(c_{22}/c_{23}-c_{32}/c_{33}\right)-$$
$$-\ \bar{q}_2/c_{23}G\left(c_{22}/c_{23}-c_{32}/c_{33}\right)$$
$$K_3=\bar{q}_4/G+c_{42}K_2+C_{43}K_1;$$
$$K_4=b_{22}K_2+b_{23}K_1;\qquad K_5=b_{32}K_2+b_{33}K_1$$

C. Irregular(Non symmetric) section

We note that in non-symmetric sections there are interactions between all the four strain modes such that the elements of (5a) and (5b) are non zero except matrix **S,** where only S_{33} has a non zero value as explained earlier.

Expanding (5a) and (5b) in line with above observations we obtain:

$$ka_{11}U_1''+ka_{12}U_2''+ka_{13}U_3''-b_{11}U_1-b_{12}U_2-b_{13}U_3$$
$$-c_{11}V_1'-c_{12}V_2'-c_{13}V_3'-c_{14}V_4'=0 \quad (25)$$
$$ka_{21}U_1''+ka_{22}U_2''+ka_{23}U_3''-b_{21}U_1-b_{22}U_2-b_{23}U_3$$
$$-c_{21}V_1'-c_{22}V_2'-c_{23}V_3'-c_{24}V_4'=0 \quad (26)$$
$$ka_{31}U_1''+ka_{32}U_2''+ka_{33}U_3''-b_{31}U_1-b_{32}U_2-b_{33}U_3$$
$$-c_{31}V_1'-c_{32}V_2'-c_{33}V_3'-c_{34}V_4'=0 \quad (27)$$
$$c_{11}U_1'+c_{12}U_2'+c_{13}U_3'+r_{11}V_1''+r_{12}V_2''$$
$$+r_{13}V_3''+r_{14}V_4''=-q_1/G \quad (28)$$
$$c_{21}U_1'+c_{22}U_2'+c_{23}U_3'+r_{21}V_1''+r_{22}V_2''$$
$$+r_{23}V_3''+r_{24}V_4''=-q_2/G \quad (29)$$
$$c_{31}U_1'+c_{32}U_2'+c_{33}U_3'+r_{31}V_1''+r_{32}V_2''+r_{33}V_3''$$
$$+r_{34}V_4''-ks_{33}V_3=-q_3/G \quad (30)$$
$$c_{41}U_1'+c_{42}U_2'+c_{43}U_3'+r_{41}V_1''+r_{42}V_2''+r_{43}V_3''$$
$$+r_{44}V_4''=-q_4/G \quad (31)$$

Equations (25) to (31) contain seven unknown parameters, U_1 ,U_2, U_3 and V_1, V_2, V_3, V_4, which can, by elimination of U_1, U_2, and U_3 be reduced to four basic equations containing V_1, V_2, V_3, and V_4. Thus the coupled differential equations of equilibrium for non symmetric sections are of the form:

$$A_1V_1^{iv} + A_2V_2^{iv} + A_3V_3^{iv} + A_4V_4^{iv} + A_5V_1^{"} + A_6V_2^{"} + A_7V_3^{"} + A_8V_4^{"} + A_9V_3 = \mu_2 K_4 \quad (32)$$

$$B_1V_1^{iv} + B_2V_2^{iv} + B_3V_3^{iv} + B_4V_4^{iv} + B_5V_1^{"} + B_6V_2^{"} + B_7V_3^{"} + B_8V_4^{"} + B_9V_3 = \varepsilon_2 K_4 \quad (33)$$

$$C_1V_1^{iv} + C_2V_2^{iv} + C_3V_3^{iv} + C_4V_4^{iv} + C_5V_1^{"} + C_6V_2^{"} + C_7V_3^{"} + C_8V_4^{"} + C_9V_3 = \gamma_2 K_4 \quad (34)$$

$$D_1V_1^{iv} + D_2V_2^{iv} + D_3V_3^{iv} + D_4V_4^{iv} + D_5V_3^{"} = 0 \quad (35)$$

Simplification and integration of these equations for flexural-torsional-distortional analysis of asymmetric box girder structures are treated in a separate paper.

VI. FLEXURAL-TORSIONAL- DISTORTIONAL ANALYSIS

In this section the solutions of the differential equations of equilibrium are obtained for single cell rectangular box girder structure and single cell trapezoidal box girder structure. Live loads were considered according to AASHTO-LRFD following the HL-93 loading [16]: uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110 KN axles. The loads were positioned at the outermost possible location to generate the maximum torsional effects. A 50m span, simply supported bridge deck structure was considered. The torsional loads obtained are as follows:

$\bar{q}_3 = q_3 * b_t = 157.16KN \quad \bar{q}_4 = q_4 * b_t = 1446.505KN$

where b_t is the width of the top flange.

A. Single Cell Rectangular (Doubly Symmetric) Section

The differential equation governing flexural-torsional-distortional equilibrium for doubly symmetric sections are

$$\beta_1 V_4" - \gamma_1 V_3 = -K_1$$
$$V_3^{iv} + \alpha_1 V_4^{iv} + \beta_2 V_4" = K_2 \quad (36)$$

The coefficients are as follows;

$a_{22} = 27.994, \quad b_{22} = c_{22} = r_{22} = 2.880$

$a_{33} = 6.721,\ b_{33} = c_{33} = 3.737;\ c_{43} = r_{43} = 3.732$

$r_{44} = 22.032; \quad s_{33} = 1.062*10^{-3}$

$\upsilon = 0.25; \quad E = 24*10^9\ N/m^2;\ G = 9.6*10^9\ N/mm^2$

$\bar{q}_2 = 0.00KN,\ \bar{q}_3 = 620KN,\ \ \bar{q}_4 = 1462.62KN$

$\alpha_1 = r_{44}/c_{43} = 5.904;$

$\beta_1 = (r_{34}/c_{33}) - (r_{44}/c_{43}) = -4.905$

$\beta_2 = (c_{43}^2 - b_{33}r_{44})/ka_{33}c_{43} = -1.0905;$

$\gamma_1 = ks_{33}/c_{33} = 7.106*10^{-4}$

$K_1 = (\bar{q}_3/c_{33}G) - (\bar{q}_4/c_{43}G) = -2.2856*10^{-5};$

$K_2 = b_{33}\bar{q}_4/a_{33}c_{43}E = 8.634*10^{-6}$

Substituting the coefficients into (36) we obtain

$$4.905V_4" + 7.106*10^{-4}V_3 = K_1 \quad (37)$$
$$V_3^{iv} + 5.904V_4^{iv} - 1.0905V_4" = K_2$$

Integrating by method of trigonometric series with accelerated convergence we obtain;

$$V_3(x) = 1.946*10^{-2}\,Sin(\pi x/50);$$
$$V_4(x) = 1.896*10^{-3}\,Sin(\pi x/50) \quad (38)$$

B. Single Cell Trapezoidal (Mono Symmetric) Section

The governing equations of equilibrium are

$$\beta_4 V_2" + \beta_5 V_3" + \beta_6 V_4" - \gamma_1 V_3 = -K_3 \quad (39)$$

$$\alpha_3 V_2^{iv} + \alpha_5 V_3^{iv} + \alpha_6^{iv} V_4^{iv} - \beta_7 V_2" - \beta_8 V_3" - \beta_9 V_4" + \gamma_2 V_3 = K_4 \quad (40)$$

$$\alpha_7 V_2^{iv} + \alpha_8 V_3^{iv} + \alpha_9 V_4^{iv} - \beta_{10} V_2" - \beta_{11} V_3" - \beta_{12} V_4" + \gamma_3 V_3 = K_5 \quad (41)$$

The relevant Vlasov's coefficients are as follows.

$a_{22} = 25.05,\ a_{23} = -0.270,\ a_{33} = 0.757,$

$b_{22} = c_{22} = r_{22} = 2.982$

$b_{23} = c_{23} = r_{23} = -0.153$

$b_{33} = c_{33} = r_{33} = 1.407, \quad r_{44} = 14.616$

$c_{24} = c_{42} = r_{24} = r_{42} = -2.515$

$c_{34} = c_{43} = r_{34} = r_{43} = 1.265$

$s_{33} = 1.8195*10^{-4}$

The coefficients of the governing equations are as follows.

$\alpha_1 = -1;\ \alpha_2 = 0.802;\ \alpha_3 = -1.6679*10^{-5}$

$\alpha_4 = -62.625;\ \alpha_5 = 0.675;\ \alpha_6 = 50.7734;$

$\alpha_7 = 0.675;\ \alpha_8 = -1.893;\ \alpha_9 = -2.078$

$\beta_1 = -1;\ \beta_2 = -0.812;\ \beta_3 = -3.251*10^{-4}$

$\beta_4 = 0;\ \beta_5 = 0;\ \beta_6 = 11572;$

$\beta_7 = 0;\ \beta_8 = -7.89*10^{-4};\ \beta_9 = -1.266;$

$\beta_{10} = 0\ \ \beta_{11} = 0;\ \beta_{12} = -1.90*10^{-4}$

$\gamma_1 = -3.532*10^{-4};\ \gamma_2 = 4.8*10^{19}$

$\gamma_3 = -4.350*10^{-4}$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$K_3 = 1.7115*10^{-4}$

$K_4 = -2.123*10^{-6}$ **;** $K_5 = -2.046*10^{-5}$

$\bar{q}_2 = 0.0KN, \quad \bar{q}_3 = 154.58KN, \quad \bar{q}_4 = 1446.505KN$

$E = 24*10^9\,N/m^2, \quad G = 9.6*10^9\,N/m^2, \quad k = 2.5$

Substituting the coefficients and the constants into (39) to (41) we obtain:

$$11.572V_4'' + 3.532*10^{-4}V_3 = -1.7115*10^{-4} \qquad (42)$$

$$-62.625V_2^{iv} + 0.675V_3^{iv} + 50.773V_4^{iv} + 7.89*10^{-4}V_3 + 1.266V_4'' = -2.123*10^{-6} \qquad (43)$$

$$0.675V_2^{iv} - 1.893V_3^{iv} - 2.078V_4^{iv} + 5.820*10^{-4}V_3'' + 1.90*10^{-4}V_4'' - 4.35*10^{-4}V_3 = -2.046*10^{-5} \qquad (44)$$

Integrating by method of trigonometric series with accelerated convergence we have,

$$V_2(x) = -3.287*10^{-2}\,Sin(\pi x/50)$$
$$V_3(x) = 4.282*10^{-2}\,Sin(\pi x/50) \qquad (45)$$
$$V_4(x) = 4.077*10^{-3}\,Sin(\pi x/50)$$

VII. DISCUSSION OF RESULTS

The derived equations for flexural-torsional-distortional analysis of box girder structures with various shapes are shown in Table 2. For rectangular and trapezoidal box sections the derived equations are independent of pure bending deformation V_1. Thus, flexural-torsional analysis of straight rectangular and trapezoidal box girder cross sections does not require evaluation of the dead load of the structure as only the imposed loads contribute to the torsional-distortional deformations of the box girder frame.

For rectangular box girder cross section, strain mode 2 (flexure), does not interact with strain modes 3 (distortion) and 4 (torsion), hence the value of V_2 does not appear in the equilibrium equations. However for trapezoidal (mono symmetric) box girder cross section, strain mode 2 has interactions with strain modes 3 and 4 resulting to the coupling of the derived equation for torsional-distortional analysis of such box girder structures as can be seen from Table 2.

For irregular cross sectional shapes the equations of equilibrium are coupled and they contain all cross sectional deformation parameters (V_1 , V_2, V_3, and V_4) as unknown.

Fig. 6 shows the variation of torsional and distortional deformations along the length of a 50m simply supported rectangular box girder frame. The flexural displacement was zero as earlier explained. The maximum mid-span distortional displacement (19.46mm), was about ten times that for torsional displacement (1.90mm).

In Fig. 7 the variation of flexural, torsional, and distortional displacements along the length of a 50m simply supported trapezoidal box girder frame is shown. .The maximum mid-span deformations are 32.87mm for flexure, 42.82mm for distortion and 4.89mm for torsion. Thus distortional deformation was about ten times that of torsional deformation and flexural deformation was eight times that of torsional deformation.

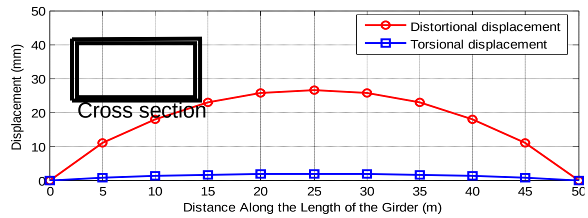


Fig.6 Variation of torsional and distortional displacements along the length of the girder

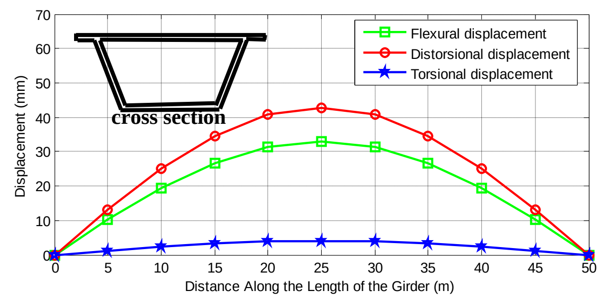


Fig.7. Variation of flexural, torsional and distortional displacements along the length of the girder

VIII. CONCLUTION

Three sets of differential equations of equilibrium were established for flexural-torsional-distortional analysis of rectangular, trapezoidal and irregular shaped box girder structures, using Vlasov's theory. The equations are appropriate for analysis of doubly symmetric, mono symmetric and asymmetric box girder cross sections.

Application of these equations in the analysis of doubly symmetric and mono symmetric box girder structures indicate that maximum distortional deformation was about ten times that of St Venant torsion.

Table.2: Summary of flexural-torsional-distortional differential equations of equilibrium

(a) Doubly symmetric section	(b) Mono symmetric section	(c) Asymmetric section
$V_2^{iv} = K_0$ (1) $\beta_1 V_4'' - \gamma_1 V_3 = -K_1$ (2) $V_3^{iv} - \alpha_1 V_4^{iv} + \beta_2 V_4'' = K_2$ (3)	$\beta_4 V_2'' + \beta_5 V_3'' + \beta_6 V_4'' - \gamma_1 V_3 = -K_3$ (1) $\alpha_4 V_2^{iv} + \alpha_5 V_3^{iv} + \alpha_6 V_4^{iv} - \beta_7 V_2''$ $-\beta_8 V_3'' - \beta_9 V_4'' + \gamma_2 V_3 = K_4$ (2) $\alpha_7 V_2^{iv} + \alpha_8 V_3^{iv} + \alpha_9 V_4^{iv} - \beta_{10} V_2''$ $-\beta_{11} V_3'' - \beta_{12} V_4'' + \gamma_3 V_3 = K_5$ (3)	$A_1 V_1^{iv} + A_2 V_2^{iv} + A_3 V_3^{iv} + A_4 V_4^{iv} + A_5 V_1''$ $+A_6 V_2'' + A_7 V_3'' + A_8 V_4'' + A_9 V_3 = \mu K_4$ (1) $B_1 V_1^{iv} + B_2 V_2^{iv} + B_3 V_3^{iv} + B_4 V_4^{iv} + B_5 V_1''$ $+B_6 V_2'' + B_7 V_3'' + B_8 V_4'' + B_9 V_3 = \varepsilon_2 K_4$ (2) $C_1 V_1^{iv} + C_2 V_2^{iv} + C_3 V_3^{iv} + C_4 V_4^{iv} + C_5 V_1''$ $+C_6 V_2'' + +C_7 V_3'' + C_8 V_4'' + C_9 V_3 = \tau_2 K_4$ (3) $D_1 V_1^{iv} + D_2 V_2^{iv} + D_3 V_3^{iv} + D_4 V_4^{iv} + D_5 V_3'' = 0$ (4)

REFERENCES

1. V.Z.Vlasov, Thin-*walled space structures* (Gosstrojizdat, Moscow, 1958).
2. C.A. Chidolue, Torsional-distortional analysis of thin-walled box girder bridges using Vlasov's theory, Ph.D. thesis, University of Nigeria, Nsukka, 2012.
3. Y.T. Hsu, C.C. Fu and D.R. Shelling, EBEF method for distortional analysis of steel box girder bridges, *Journal of Structural Engineering, 121(3) 1995, 557-566*
4. Z. Fan, and T.A. Helwig, Distortional loads and brace forces in steel box girders, *Journal of. Structural Engineering, 128(6)*, 2002, 710-718.
5. K.M. Sennah, and J.B. Kennedy, Literature review in analysis of box girder bridges, *Journal of Bridge Engineering, 7 (2)*, 2002, 134-140.
6. A.M. Okeil, and S. El-Tawil, Warping stresses in curved box girder bridges: case study, *Journal of Bridge Engineering, 9(5)* , 2004, 487- 496.
7. J. Paavola, *Study of curved Thin-Walled Girders*, doctoral diss., Helsinki University of Technology, Espoo, Finland, PhD, 1990.
8. A.G. Razaqpur, and H.G. Li, Analysis of multi-branch multi-cell box girder bridges, *Proc.3rd Int. Conf. on Short and Medium Span Bridges*, Toronto, 1990, 153-164
9. C.C. Fu, and Y.T. Hsu, The Development of an Improved Curvilinear Thin-Walled Vlasov Element, *Computational Structures Journal, 54(1),* 1995, 147-159.
10. A. Tesar, Shear Lag in the Behaviour of Thin-Walled Box Bridges, *Comput. Struct. Journal, 59(4),* 1996, 607-612.
11. N.N. Osadebe and C.A. Chidolue, Torsional-distorsional response of thin-walled mono symmetric box girder structures, *International journal of engineering research and applications*, 2 (3), 2012, 814-821
12. N.N. Osadebe and C.A. Chidolue, Response of double cell mono symmetric box girder structure to torsional-distortional deformations, *Intrnational journal of engineering and advanced technology*, 1 (4), 2012, 285-292.
13. Ezeh J.C., Buckling behaviour of axially compressed multi cell doubly symmetric thin-walled column using Vlasov's theory, *International journal of engineering*, 4 (2), 2010, 179-194
14. C.P. Varbanov, *Theory of elasticity* (Technika Press Sofia, 4th Edition, 1976).
15. L. Elsgolt, *Differential Equations and the Calculus of Variation,* (MIR publishers, Mosco,1980, translated from the Russian by George Yankovsky.)
16. American Association of State Highway and Transportation Officials (AASHTO), *Load and Resistance Factor Design,* LRFD, Bridge Design Specifications Washington , D.C, 1998.
17. C.A. Chidolue and N.N. Osadebe, Flexural-torsional behaviour of thin-walled mono symmetric box girder structures, International Journal of Engineering Sciences and Emerging Technologies, 2 (2), 2012, 11-19
18. N.N.Osadebe and C.A. Chidolue, Flexural-distortional behaviour of thin-walled mono symmetric box girder structure, International Journal of Engineering Research and Technology, to be published.

AUTHORS PROFILE

Chinenye Alfred Chidolue, obtained B. Eng (Civil) degree from the University of Zambia , M. Eng degree (Civil and Structural) from University of Sheffield and Ph. D. degree in Structural Engineering is being awaited from University of Nigeria, Nsukka. He is currently a Senior lecturer in Civil Engineering Department, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria

Chukwura Henry Aginam, obtained B. Eng (Civil) degree from the University of Nigeria , M. Sc. Degree (Structures) from University of Lagos, Nigeria and Ph. D. degree in Structural Engineering from University of Nigeria, Nsukka. He is currently a Senior lecturer in Civil Engineering Department, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria