Response of Double Cell Mono Symmetric Box Girder Structure to Torsional-Distortional Deformations

N. N. Osadebe, C. A. Chidolue

Abstract- In this paper the effect of middle web member on torsional-distortional deformations of a double cell mono symmetric box girder structure is examined. First, the torsional and distortional deformations of a single cell mono symmetric box girder structure were examined using a single span, simply supported bridge structure on the bases of Vlasov's theory. By introducing a middle (vertical) web member on the single cell mono symmetric box girder section a double cell mono symmetric box girder structure of the same overall cross sectional dimensions was obtained. The torsional and distortional deformations of the double cell cross sectional profile were also evaluated and compared with those of the mono symmetric cross sectional profile. Results show that the introduction of the middle web member to obtain the double cell mono symmetric box girder structure reduced the distortional deformation by 118% and increased the torsional deformation by 14%

Keywords:- box girder, distortion, mono symmetric, thin-walled, torsion, Vlasov's theory.

I. INTRODUCTION

Basically, a curved structural element has two interacting forces: bending and torsion. Knowledge of this interaction leads to a successful design of the element. The study of curved elements offers only one instance where torsional and bending forces simultaneously occur. In other instances, e.g., straight girders (in either bridges or buildings) may develop such forces and thus require appropriate analysis and design. This situation arises where the load is eccentric with respect to the girder axis or shear centre. The engineer is then forced to determine the shear centre, resolve the forces into appropriate bending and torsional forces and then determine the stresses and deformations.

The torsional response of structural elements can be classified into two categories: pure torsion and warping torsion or bimoment. Most structural engineers are familiar with the concept of pure torsion as this type of torsion is studied in strength of materials courses. The second type of torsion, warping torsion and distortion are probably new terms and phenomena which need to be fully investigated in order to avoid their undesired effects, particularly on box girder structures.

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The general theories described by Heins [1] relative to straight and curved girders assume thin walled sections which may be open or closed (box section). Development of general equations was based on prismatic girders (solid beams). Their solution then permits a proper engineering examination of box girder elements when subjected to vertical torsional loads.

The application of cross sectional deformation equations formulated by Vlasov [2] and Dabrowski [3], modified by Varbanov [4], has opened a new way to analyze the torsional and distortional effects of loads on such girders. The purpose of this work is to examine the effect of introducing a middle (vertical) web member on the torsional and distortional deformations of a mono symmetric box girder section.

II. REVIEW OF PAST WORK

The curvilinear nature of box girder bridges along with their complex deformation patterns and stress fields have led designers to adopt approximate and conservative methods for their analysis and design. Recent literatures, Hsu et al [5], Fan and Helwig [6], Sennah and Kennedy [7], on straight and curved box girder bridges deal with analytical formulations to better understand the behaviour of these complex structural systems. Few authors, Okil and El-tawil [8], Sennah and Kennedy [7], have undertaken experimental studies to investigate the accuracy of existing methods. Before the advent of Vlasov's theory of thin-walled beams[2] the conventional method of predicting warping and distortional stresses is by beam on elastic foundation (BEF) analogy. This analogy ignores the effect of shear deformations and takes no account of the cross sectional deformations which are likely to occur in a thin walled box girder structure

Several investigators; Bazant and El-Nimeiri [9], Zhang and Lyons [10], Boswell and Zhang [11], Usuki [12], Waldron [13], Paavola [14], Razaqpur and Lui [15], Fu and Hsu [16], Tesar [17], have combined thin-walled beam theory of Vlasov and the finite element technique to develop a thin walled box element for elastic analysis of straight and curved cellular bridges.

Various theories were postulated by different authors examining methods of analysis, both classical and numerical. A few others however carried out tests on prototype models to verify the authenticity of the theories. At the end of it all, it was concluded that Vlasov's theory captures all peculiarities of cross sectional deformation such as warping, torsion,

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distortion etc, and is therefore adopted in this work.

III. VLASOV'S STRESS – STRAIN RELATIONS

The longitudinal warping and transverse (distortional) displacements given by Vlasov [2] are:

$$u(x,s)=U(x)\varphi(s)\,;\quad v(x,s)=V(x)\psi(s) \qquad (1)$$

These displacements may be represented in series form as:

$$u(x,s)=\sum_{i=1}^{m}U_i(x)\varphi_i(s)$$
$$v(x,s)=\sum_{k=1}^{n}V_k(x)\psi_k(s) \qquad (2)$$

where, $U_i(x)$ and $V_k(x)$ are unknown functions which express the laws governing the variation of the displacements along the length of the space frame. $\varphi_i(s)$ and $\psi_k(s)$ are elementary displacements of the strip frame, respectively out of the plane (m displacements) and in the plane (n displacements). These displacements are chosen among all displacements possible, and are called the generalized strain coordinates of a strip frame.

From the theory of elasticity the strains in the longitudinal and transverse directions are given by;

$$\frac{\partial u(x,s)}{\partial x}=\sum_{i=1}^{m}U_i'(x)\varphi_i(s)$$
$$\frac{\partial v(x,s)}{\partial x}=\sum_{k=1}^{n}V_k'(x)\psi_k(s) \qquad (3)$$

The expression for shear strain is $\gamma(x,s)=\frac{\partial u}{\partial s}+\frac{\partial v}{\partial x}$ or

$$\gamma\left(x,s\right)=\sum_{i=1}^{m}\varphi_i'(s)U_i(x)+\sum_{k=1}^{n}\psi_k(s)V_k'(x) \qquad (4)$$

Using the above displacement fields and basic stress-strain relationships of the theory of elasticity the expressions for normal and shear stresses become,

$$\sigma(x,s)=E\frac{\partial u(x,s)}{\partial x}=E\sum_{i=1}^{m}\varphi_i(s)U_i'(x) \qquad (5)$$

$$\tau(x,s)=G\gamma(x,s)$$
$$=G\left[\sum_{i=1}^{m}\varphi_i'(s)U_i(x)+\sum_{k=1}^{n}\psi_k(s)V_k'(x)\right] \qquad (6)$$

Transverse bending moment generated in the box structure due to distortion is given by $M\left(x,s\right)=\sum_{k=1}^{n}M_k(s)V_k(x)$

(7)

where $M_k(s)$ = bending moment generated in the cross sectional frame of unit with due to a unit distortion, V(x) = 1

IV. ENERGY FORMULATION OF THE EQUILIBRIUM EQUATIONS

The potential energy of the box structure under the action of a distortional load of intensity q is given by:

$$\Pi=U+W_E \qquad (8)$$

where,

Π = the total potential energy of the box structure,

U = Strain energy,

W_E = External potential or work done by the external loads.

From strength of material, the strain energy of a structure is given by

$$U=\frac{1}{2}\int_L\int_S\left[\begin{array}{l}\left(\sigma^2(x,s)/E+\tau^2(x,s)/G\right)t(s)\\+M^2(x,s)/EI_s-qv(x,s)\end{array}\right]dxds \qquad (9)$$

Work done by external load is given by

$$W_E=qv(x,s)dxds=\int_s\int_x q\sum V_h(x)\varphi_h(s)dsdx$$

$$=\int_x\sum q_hV_hdx \qquad (10)$$

Substituting (9) and (10) into (8) we obtain,

$$\Pi=\frac{1}{2}\left[\int_L\int_S\begin{array}{l}\left[\sigma^2(x,s)/E+\tau^2(x,s)/G\right]t(s)\\+M^2(x,s)/EI_s-qv(x,s)\end{array}\right]dxds$$

(11)

where,

$\sigma(x,s)$ = Normal stress

$\tau(x,s)$ = Shear stress

$M(x,s)$ = Transverse distortional bending moment

q = Line load per unit area applied in the plane of the plate

$I_{(s)}=t^3(s)/12(1-\nu^2)$ = Moment of inertia of plate

E = Modulus of elasticity

G = Shear modulus

ν = poisson ratio

t = thickness of plate

Substituting (1), (5), (6), and (7) into (11), noting that $t(s)ds=dA$, we obtain the potential energy of the box structure, after simplification, as follows.

$$\Pi=\frac{E}{2}\sum a_{ij}U_i'(x)U_j'(x)dx+$$
$$+\frac{G}{2}\left[\sum b_{ij}U_i(x)U_j(x)+\sum c_{kj}U_k(x)V_j'(x)\right]dx$$

+

$$\frac{G}{2}\left[\sum c_{ih}U_i(x)V_h{}'(x)+\sum r_{kh}V_k{}'(x)V_h{}'(x)\right]dx+$$
$$+\frac{E}{2}\sum s_{hk}V_k(x)V_h(x)dx-\sum q_h V_h dx \qquad (12)$$

where the (Vlasov's) coefficients are defined as follows:

$$a_{ij}=a_{ji}=\int \varphi_i(s)\varphi_j(s)dA \qquad (a)$$
$$b_{ij}=b_{ji}=\int \varphi_i^{'}(s)\varphi_j^{'}(s)dA \qquad (b)$$
$$c_{kj}=c_{jk}=\int \varphi_k^{'}(s)\psi_j(s)dA \qquad (c)$$
$$c_{ih}=c_{hi}=\int \varphi_i^{'}(s)\psi_k(s)dA \qquad (d) \qquad (13)$$
$$r_{kh}=r_{hk}=\int \psi_k(s)\psi_h(s)dA; \qquad (e)$$
$$s_{kh}=s_{hk}=\frac{1}{E}\int\frac{M_k(s)M_h(s)}{EI_{(s)}}ds \qquad (f)$$
$$q_h=\int q\psi_h ds \qquad (g)$$

The governing equations of torsional-distortional equilibrium are obtained by minimizing the energy functional (12), with respect to its functional variables u(x) and v(x) using Euler Lagrange technique, Elgolts [18]. Minimizing with respect to u(x) we obtain:

$$k\sum_{i=1}^{m}a_{ij}U_i{}''(x)-\sum_{i=1}^{m}b_{ij}U_i(x)-\sum_{k=1}^{n}c_{kj}V_k{}'(x)=0 \qquad (14)$$

Minimizing with respect to v(x) we have:

$$\sum_{i=1}^{m}c_{ih}U_i{}'(x)+\sum_{k=1}^{n}r_{kh}V_k{}''(x)-$$
$$-k\sum_{k=1}^{n}s_{hk}V(x)+(1/G)\sum_{h=1}^{n}q_h=0 \qquad (15)$$

where $\kappa=E/G=2(1+\nu)$

Equations (14) and (15) are Vlasov's generalized differential equations of distortional equilibrium for a box girder. They are presented in matrix form as follows:

$$\kappa\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}\begin{Bmatrix}U_1''\\U_2''\\U_3''\end{Bmatrix}-\begin{bmatrix}b_{11}&b_{12}&b_{13}\\b_{21}&b_{22}&b_{23}\\b_{31}&b_{32}&b_{33}\end{bmatrix}\begin{Bmatrix}U_1\\U_2\\U_3\end{Bmatrix}$$
$$-\begin{bmatrix}c_{11}&c_{12}&c_{13}&c_{14}\\c_{21}&c_{22}&c_{23}&c_{24}\\c_{31}&c_{32}&c_{33}&c_{34}\end{bmatrix}\begin{Bmatrix}V_1'\\V_2'\\V_3'\\V_4'\end{Bmatrix}=0 \qquad (16a)$$

$$\begin{bmatrix}c_{11}&c_{12}&c_{13}\\c_{21}&c_{22}&c_{23}\\c_{31}&c_{32}&c_{33}\\c_{41}&c_{42}&c_{43}\end{bmatrix}\begin{Bmatrix}U_1'\\U_2'\\U_3'\end{Bmatrix}-\kappa\begin{bmatrix}s_{11}&s_{12}&s_{13}&s_{14}\\s_{21}&s_{22}&s_{23}&s_{24}\\s_{31}&s_{32}&s_{33}&s_{34}\\s_{41}&s_{41}&s_{43}&s_{44}\end{bmatrix}\begin{Bmatrix}V_1\\V_2\\V_3\\V_4\end{Bmatrix}+$$
$$+\begin{bmatrix}r_{11}&r_{12}&r_{13}&r_{14}\\r_{21}&r_{22}&r_{23}&r_{24}\\r_{31}&r_{32}&r_{33}&r_{34}\\r_{41}&r_{42}&r_{43}&r_{44}\end{bmatrix}\begin{Bmatrix}V_1''\\V_2''\\V_3''\\V_4''\end{Bmatrix}=\frac{1}{G}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix} \qquad (16b)$$

V. GENERATION OF STRAIN MODES AND EVALUATION OF VLASOV'S COEFFICIENTS

Vlasov's coefficients of differential equations of equilibrium (13), which involve a combination of elementary displacements and their derivatives may be obtained by consideration of the box girder cross sections, Figs. 1(a) and 2(a), as strip frames.

The single cell mono-symmetric strip frame in Fig. 1(a) has four degrees of freedom in the longitudinal direction and four in the transverse direction. From (2), where in this case m = 4 and n = 4, it follows that we have sixteen displacement quantities to compute and hence, sixteen differential equations of distortional equilibrium will be required. For multi-celled profiles the number of degrees of freedom will increase and so will the number of differential equations require to solve for the displacement quantities, and this can be quite cumbersome. The application of Vlasovs generalized strain modes as modified by Varbanov [4], reduces the number of displacement quantities and hence the differential equations of equilibrium required to solve for them to seven, irrespective of the number of degrees of freedom possessed by the structure.

In the generalized strain modes, there are three strain fields in the longitudinal direction, φ_1, φ_2, and φ_3. Thus, from (2) we have,

$$u(x,s)=U_1(x)\varphi_1(s)+U_2(x)\varphi_2(s)+U_3(x)\varphi_3(s) \text{ or}$$
$$u(x,s)=\sum_{i=1}^{3}U_i(x)\varphi_i(s) \qquad (17)$$

In the transverse direction, four strain modes are also recognized; ψ_1, ψ_2, ψ_3, and ψ_4. From (2) we have:

$$v(x,s)=V_1(x)\psi_1(s)+V_2(x)\psi_2(s)+V_3(x)\psi_3(s)+$$
$$+V_4(x)\psi_4(s) \quad \text{or}$$
$$v(x,s)=\sum_{k=1}^{4}V_k(x)\,\psi_k(s) \qquad (18)$$

where φ_1 = out of plane displacement parameter when the load is acting (vertically) normal to the top flange of the girder, i.e., bending is about horizontal axis.

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φ_2 = Out of plane displacement parameter when the load is acting tangential to the plane of the flanges, i.e., bending is about vertical axis.

φ_3 = Out of plane displacement parameter due to distortion of the cross section, i.e., the warping function .

ψ_1 = In-plane displacement parameter due to the load giving rise to φ_1

ψ_2 = In-plane displacement parameter due to the load giving rise to φ_2

ψ_3 = In-plane displacement parameter due to the distortion of the cross section, i.e., non uniform torsion.

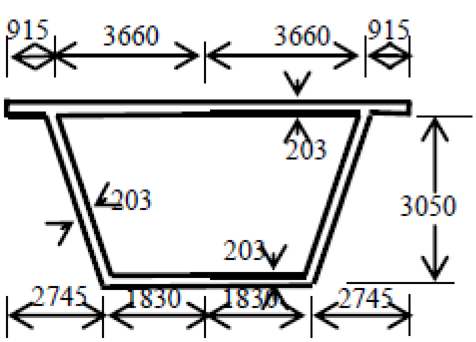
ψ_4 = In-plane displacement function due to pure rotation or Saint Venant torsion of the cross section.

VI. VI. STRAIN MODE DIAGRAMS

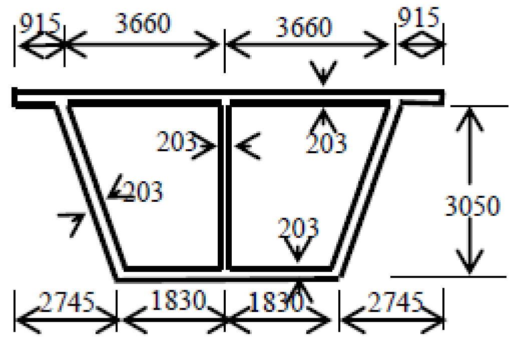
The procedure for obtaining the strain mode diagrams for mono symmetric cross sections is given in literatures, Osadebe and Chidolue [20], Rekach [21]

Fig. 1(b) shows a double cell mono symmetric box girder frame used for numerical analysis. The frame is obtained by introducing a vertical web member on the centre line of the frame in Fig. 1(a). Thus, the over all cross sectional dimensions of the single cell section and double cell section are the same. Fig. 2(a) to 2(g) show the strain mode diagrams for the double cell mono symmetric box girder structure. The coefficients a_{ij}, b_{ij}, c_{kj}, c_{ih}, and r_{kh} of the governing equations of equilibrium (14) and (15), are computed with the aid of Morh's integral chart using the strain mode diagrams. The summary of the coefficients for double cell mono symmetric box girder frame are on Table I.

In our paper, Osadebe and Chidolue [20], the strain mode diagrams and Vlasov's coefficients for the single cell mono symmetric frame in Fig. 1(a) are given.

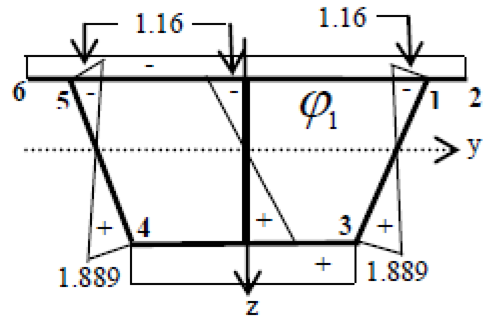


(a) Single cell box girder section

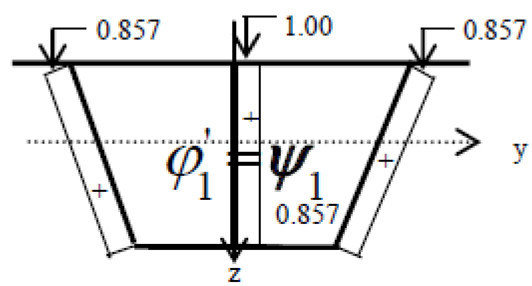


(b) Double cell box girder section

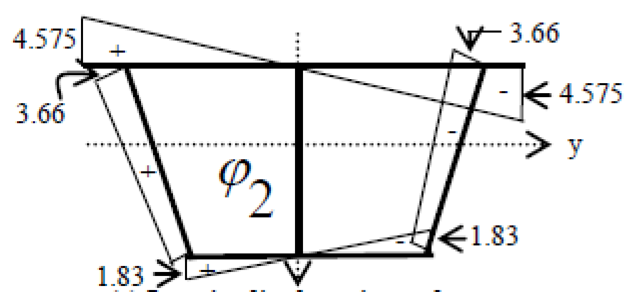
Fig.1 Mono symmetric box girder frames



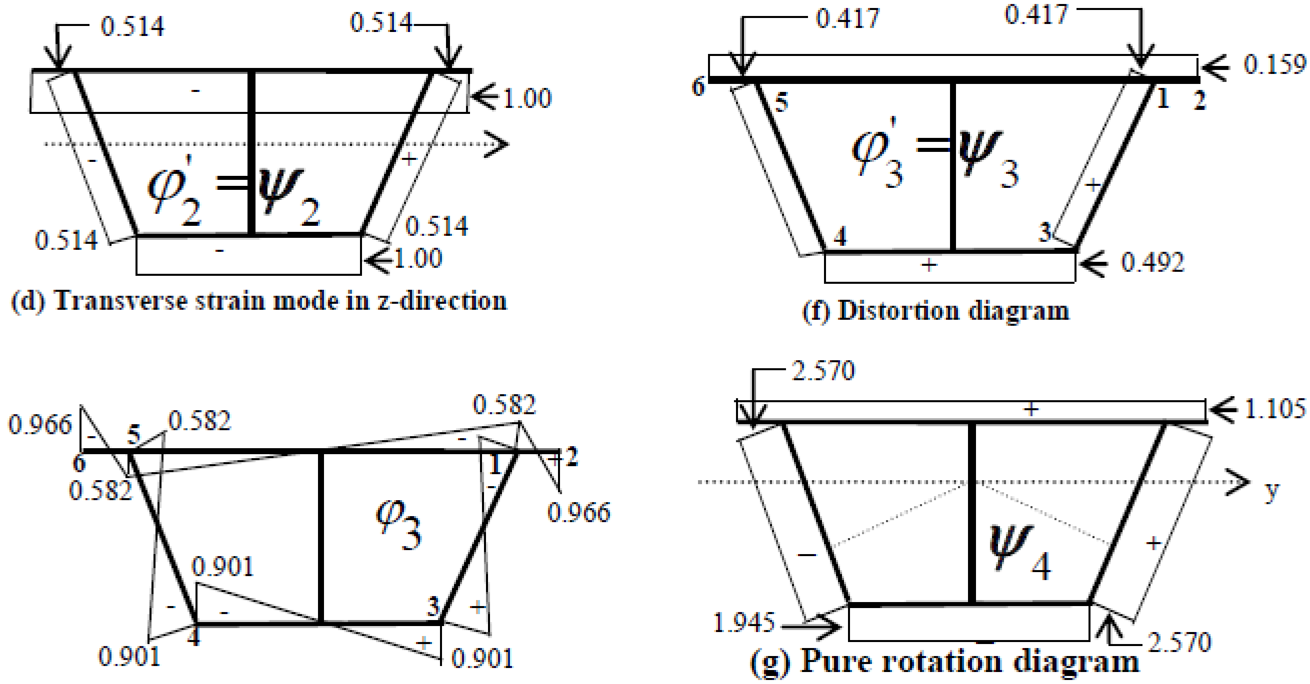
(a) Longitudinal strain mode diagram (Bending about y-y Axis)



(b) Transverse stain mode in y-y direction



(c) Longitudinal strain mode (Bending about z-z axis)



(d) Transverse strain mode in z-direction

(f) Distortion diagram

(e) Warping function diagram

(g) Pure rotation diagram

Fig. 2 Generalized strain modes for double cell mono-symmetric box girder frame

Table I: Summary of Vlasov's coefficients for the double cell mono symmetric frame

$a_{ij} = a_{ji}$	$b_{ij} = b_{ji}$	$c_{kj} = c_{jk}$	$c_{ih} = c_{hi}$	$r_{kh} = r_{hk}$
$a_{11} = 7.023$	$b_{11} = 1.680$	$c_{11} = 1.680$	$c_{11} = 1.680$	$r_{11} = 1.680$
$a_{12} = 0$	$b_{12} = 0$	$c_{12} = 0$	$c_{12} = 0$	$r_{12} = 0$
$a_{13} = 0$	$b_{13} = 0$	$c_{13} = 0$	$c_{13} = 0$	$r_{13} = 0$
$a_{22} = 25.073$	$b_{22} = 2.982$	$c_{22} = 2.982$	$c_{14} = 0$	$r_{14} = 0$
$a_{23} = 0.425$	$b_{23} = -0.449$	$c_{23} = -0.449$	$c_{22} = 2.982$	$r_{22} = 2.982$
$a_{33} = 0.750$	$b_{33} = 1.533$	$c_{33} = 1.533$	$c_{23} = -0.449$	$r_{23} = -0.449$
$s_{33} = 0.723 * I_s$		$c_{41} = 0$	$c_{33} = 1.533$	$r_{24} = 1.112$
		$c_{42} = 1.112$	$c_{24} = 1.112$	$r_{33} = 1.538$
		$c_{43} = 1.295$	$c_{34} = 1.295$	$r_{34} = 1.295$
				$r_{44} = 14.485$

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VII. EVALUATION OF DISTORTIONAL BENDING MOMENT COEFFICIENTS Shk

The distortional bending moments coefficients S_{hk}, given by (13f) depend on the bending deformation of the strip frame characterized by the bending moment, M_k (for k = 1, 2, 3, 4). To compute the coefficients we need to construct the diagram of the bending moments due to strain modes, $\psi_1, \psi_2, \psi_3, and\ \psi_4$,. Incidentally, ψ_1, ψ_2 and ψ_4 strain modes do not generate distortional bending moments on box girder structures as they involve pure bending and pure rotation. Only ψ_3 strain mode generates distortional bending moment which can be evaluated using the distortion diagram for the relevant cross section. Consequently the relevant expression for the coefficient becomes:

$$s_{hk} = s_{kh} = \frac{1}{E} \int_s \frac{M_3(s) M_3(s)}{EI_s} \qquad (19)$$

where $M_3(s)$ is the distortional bending moment of the relevant cross section due to strain mode 3.

The procedure for evaluation of distortional bending moments is given in literatures [20], [21]. Fig.3 shows the distortional bending moment for evaluation of S_{hk} for double cell mono symmetric frame of Fig. 1(b) using (19) and Morh's integral for diagram multiplication.

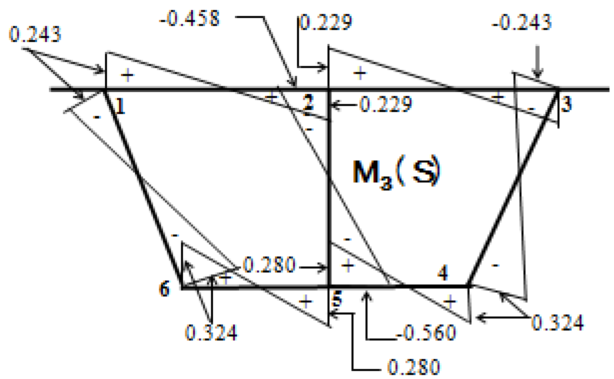

Fig.3 : Bending moment due to distortion of double cell mono-symmetric section

VIII. EQUATIONS OF EQUILIBRIUM FOR MONO SYMMETRIC CROSS SECTIONAL PROFILES

The relevant coefficients for torsional-distortional equilibrium (strain modes 3 and 4), are a_{33}, b_{33}, c_{33}, c_{34}, r_{33}, r_{34}, r_{44} and s_{33}. Substituting these into (16a) and (16b) and multiplying out we obtain;

$$ka_{33}U_3'' - b_{33}U_3 - c_{33}V_3' - c_{34}V_4' = 0 \qquad (20)$$

$$c_{33}U_3' - ks_{33}V_3 + r_{33}V_3'' + r_{34}V_4'' = -\frac{q_3}{G} \qquad (21)$$

$$c_{43}U_3' + r_{43}V_3'' + r_{44}V_4'' = -\frac{q_4}{G} \qquad (22)$$

Simplifying further we obtain the coupled differential equations of torsional-distortional equilibrium for mono symmetric sections as follows:

$$\begin{aligned} &\beta_1 V_4'' - \gamma_1 V_3 = K_1 \\ &V_3^{iv} + \alpha_2 V_4^{iv} - \beta_2 V_4'' = K_2 \end{aligned} \qquad (23)$$

where, $$\alpha_2 = \frac{r_{44}}{c_{43}}, \qquad \beta_1 = r_{34}c_{43} - c_{33}r_{44} \qquad (24a)$$

$$\beta_2 = \frac{b_{33}r_{44} - c_{34}c_{43}}{ka_{33}c_{43}}, \qquad \gamma_1 = c_{43}ks_{33} \qquad (24b)$$

$$K_1 = c_{33}\frac{q_4}{G} - c_{43}\frac{q_3}{G}; \quad K_2 = \left(\frac{b_{33}}{ka_{33}c_{43}}\right)\frac{q_4}{G} \qquad (24c)$$

IX. TORSIONAL-DISTORTIONAL ANALYSIS OF MONO SYMMETRIC PROFILES

In this section the solutions of the differential equations of equilibrium (23), are obtained for a simply supported, double cell mono symmetric box girder frame, Fig.1(b), and compared with the results of the analysis of a simply supported, single cell mono symmetric box girder structure Fig. 1(b), obtained by the authors (20). Live loads were considered according to AASHTO-LRFD following the HL-93 loading [21]. Uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110 KN axles. The loads were positioned at the outermost possible location to generate the maximum torsional effects. A 50m span, simply supported bridge deck structure was considered. The torsional loads obtained are as follows:

$\bar{q}_3 = q_3 * b_t = 157.16KN$, $\bar{q}_4 = q_4 * b_t = 1446.505KN$ where b_t is the width of the top flange.

The governing equations of equilibrium are given by (23).

A. *Single Cell Mono Symmetric Section*

The torsional-distortional differential equations of equilibrium obtained [20] for the single cell mono symmetric section are:

$$\begin{aligned} &2.371V_3^{iv} + 27.405V_4^{iv} - 18.963V_4'' = 2.120*10^{-6} \\ &-18.964V_4^{iv} - 5.503*10^{-4}V_3 = 1.9163*10^{-4} \end{aligned} \qquad (25)$$

Integrating by method of trigonometric series with accelerated convergence we have

$$\begin{aligned} &V_3(x) = 3.268*10^{-2} Sin(\pi x/50) \\ &V_4(x) = 2.80*10^{-3} Sin(\pi x/50) \end{aligned} \qquad (26)$$

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B. Double Cell Mono Symmetric Section

The relevant coefficients from Table I are,

$$a_{33} = 0.750;\; b_{33} = c_{33} = r_{33} = 1.533$$

$$c_{34} = c_{43} = r_{34} = r_{43} = 1.295;\; r_{44} = 14.485$$

$$s_{33} = 0.723 * 6.9712 * 10^{-4} = 5.04 * 10^{-4}$$

The parameters for the governing equations are,

$$\alpha_2 = k a_{33} r_{44} = 27.160$$

$$\beta_1 = r_{34} c_{43} - c_{33} r_{44} = -20.528$$

$$\beta_2 = b_{33} r_{44} - c_{34} c_{43} = 20.528$$

$$\gamma_1 = c_{43} k s_{33} = 1.632 * 10^{-3}$$

$$K_1 = - c_{43} \frac{\bar{q}_3}{G} + C_{33} \frac{\bar{q}_4}{G} = 2.613 * 10^{-4}$$

$$K_2 = b_{33} \frac{\bar{q}_4}{G} = 2.87845 * 10^{-4}$$

$$G = 9.60 * 10^{9}\, N/mm^{2}$$

Substituting these parameters into (23) we obtain:

$$2.428 V_3^{iv} + 27.16 V_4^{iv} - 20.528 V_4'' = 2.87845 * 10^{-4}$$

$$-20.528 V_4^{iv} + 1.632 * 10^{-3} V_3 = 2.613 * 10^{-4} \qquad (27)$$

Integrating by method of trigonometric series with accelerated convergence we have

$$V_3(x) = 1.500 * 10^{-2}\, Sin(\pi x/50)$$

$$V_4(x) = 3.526 * 10^{-3}\, Sin(\pi x/50) \qquad (28)$$

C. Discussion of Results

Fig.4 shows the variation of torsional and distortional displacements along the length of the single cell mono symmetric box girder as expressed by (28). Fig.5 shows the variation of torsional and distortional displacements along the length of the box girder for double cell mono symmetric profile as expressed by (29). From these figures, we observed that distortional deformations are consistently higher than torsional deformations. The maximum (mid span) torsional deformations are 3mm for single cell profile and 3.5mm for double cell profile, while the maximum (mid span) deplanation are 33mm for single cell and 15 mm for double cell.

We expected that double cell mono symmetric profile would offer greater resistance to torsional and distortional deformations than single cell mono symmetric profiles because of increase in the rigidity of the double cell structure. While this expectation was realized in the case of distortional deformation it was not so in the case of torsional deformation. This may be attributed to the effect of the interaction between torsional strain mode and distortional strain mode which resulted to the coupling of the differential equations of torsional-distortional equilibrium (23).

The effect of middle member (web) in double cell mono symmetric cross section was 120% decrease in distortional deformation and 14% increase in the torsional deformation.

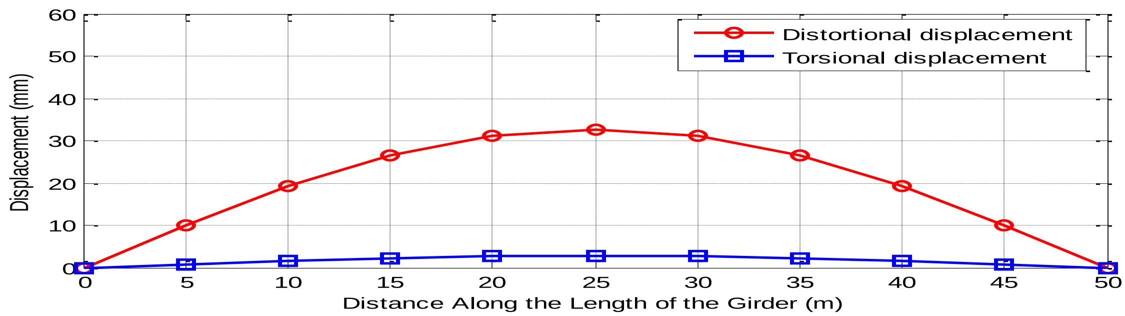

Fig.4 Variation of torsional and distortional displacements along the length of the girder (Single cell mono symmetric section

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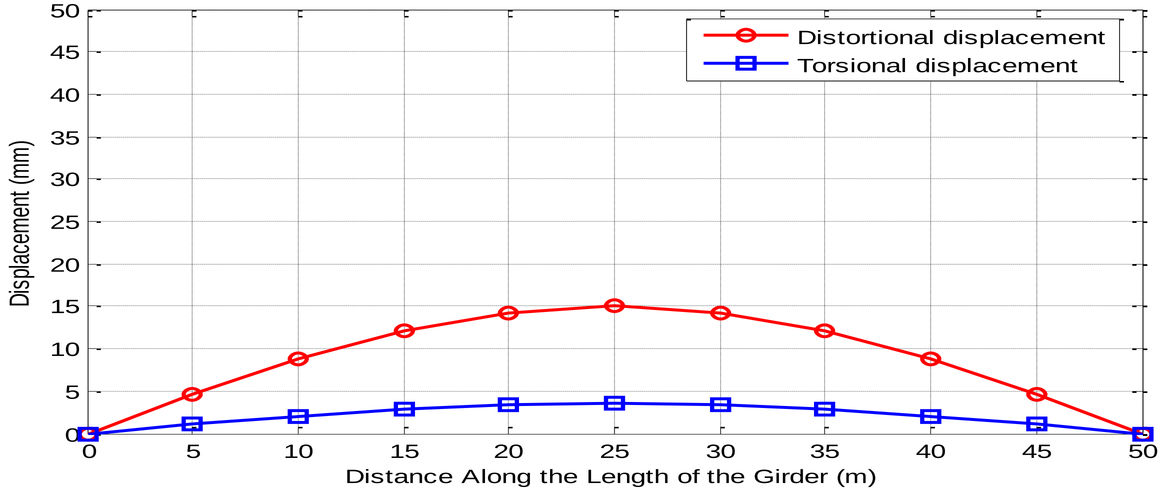

Fig.5 Variation of torsional and distortional displacements along the length of the girder (Double cell mono symmetric section)

X. CONCLUSION

Generally, the distortional deformation was found to be four to twelve times higher than the torsional deformation. The introduction of the middle (vertical) web member in the double cell mono symmetric box girder structure reduced the distortional deformation by 120% and increased the torsional deformation by 14%. Thus, use of multi-cell profiles can be a good substitute for use of diaphragms and internal bracings in mono-symmetric box girder structures.

REFERENCES

1. C.P. Heins, *Bending and torsion design in structural member* (Lexinton Books, Massachusetts, 1975).
2. V.Z.Vlasov, *Thin-walled space structures* (Gosstrojizdat, Moscow, 1958).
3. R. Dabrowski, 1968, *Curved thin-walled girder* (translation No. 144, Cement and Concrete Association, London, 1968).
4. C.P. Varbanov, *Theory of elasticity* (Technika Press Sofia, 4th Edition, 1976).
5. Y.T. Hsu, C.C. Fu and D.R. Shelling, EBEF method for distortional analysis of steel box girder bridges, *Journal of Structural Engineering, 121(3) 1995, 557-566.*
6. Z. Fan, and T.A. Helwig, Distortional loads and brace forces in steel box girders, *Journal of. Structural Engineering, 128(6),* 2002, 710-718.
7. K.M. Sennah, and J.B. Kennedy, Literature review in analysis of box girder bridges, *Journal of Bridge Engineering, 7 (2),* 2002, 134-140.
8. A.M. Okeil, and S. El-Tawil, Warping stresses in curved box girder bridges: case study, *Journal of Bridge Engineering, 9(5)* , 2004, 487-496.
9. Z.P. Bazant and M. El-Nimeiru, Stiffness method for curved box girders at initial stress, *Journal of Structural Division, 100(10),* 1974, 2071-2090.
10. S.H. Zhang and L.P.R. Lyons, Thin-walled box beam finite element for curved bridge analysis, *Comput. Struct. Journal*, 18(6), 1984, 1035-1046.
11. L.F. Boswell and S.H. Zhang, The effect of distortion in thin-walled box-spine beams, *International Journal of Solid Structures, 20(9/10),* 1984, 845-852.
12. T. Usuki, The Theory of Curved Multi-Cell Box Girder Bridges Under Consideration of Cross-Sectional Distortion, *Struct. Eng./Earthquake Eng. Journal, 4(2)*, 1987, 277-287
13. P. Waldron, The Significance of warping torsion in the design of straight concrete box-girder Bridges, *Canadian Journal of Civil Engineering, 15(5)* 1988, 879-889
14. J. Paavola, *Study of curved Thin-Walled Girders*, doctoral diss., Helsinki University of Technology, Espoo, Finland, PhD, 1990.
15. A.G. Razaqpur, and H.G. Li, Analysis of multi-branch multi-cell box girder bridges, *Proc.3rd Int. Conf. on Short and Medium Span Bridges*, Toronto, 1990, 153-164
16. C.C. Fu, and Y.T. Hsu, The Development of an Improved Curvilinear Thin-Walled Vlasov Element, *Computational Structures Journal, 54(1),* 1995, 147-159.
17. A. Tesar, Shear Lag in the Behaviour of Thin-Walled Box Bridges, *Comput. Struct. Journal, 59(4),* 1996, 607-612.
18. L. Elsgolt, *Differential Equations and the Calculus of Variation*, (MIR publishers, Mosco,1980, translated from the Russian by George Yankovsky.)
19. American Association of State Highway and Transportation Officials (AASHTO), *Load and Resistance Factor Design*, (LRFD), Bridge Design Specifications Washington , D.C, 1998).
20. N.N. Osadebe and C.A. Chidolue, Torsional-distortional response of thin-walled mono symmetric box girder structures, *International journal of engineering research and application*, submitted for publication.
21. V.G. Rekech, *Static theory of thin-walled space structures*, MIR publishers, Mosco, (1978)
22. C.A. Chidolue, *Torsional-distortional analysis of thin-walled box girder bridges using Vlasov's theory*. PhD theses, (2012), University of Nigeria, Nsukka, Nigeria