

# A Comparative Study of Image Scaling Algorithms

Kranti Kumar Jain, Tripti Sharma

**Abstract:** *In this paper, we propose comparative study of image scale retrieval scheme. To the best of our knowledge, there is less comprehensive study on large-scale evaluation. Our empirical results show that our proposed solution is able to scale for hundreds of thousands of images, which is promising for building scale systems. A comparison of various techniques for image scaling one digital image in to another is made. We will compare various image scaling techniques such as Gaussian scale mixtures in the wavelet domain, Local Wiener estimate, Multi-scale image scaling, Bayes least squares estimator, Thin Plate Spline based image scaling based on different attributes such as Computational Time, Visual Quality of image scaling obtained and Complexity involved in selection of features.*

**Keywords:** *Bayes least squares (BLS), Gaussian scale mixture (GSM), Local Wiener estimate, Multi-scale image scaling, Thin Plate Spline.*

## I. INTRODUCTION

Image scaling is the name given to an effect used frequently in film and television whereby an object – often a person – is seen to change smoothly into another object. The thing that makes image scale so striking is the seamless nature of the change. In a good scale, the object should appear well defined and physically reasonable throughout, gradually changing its shape to become something else. Image scaling was conceived in the late 1980s when computers were becoming powerful enough to handle images of the quality needed for use in the film industry. Although developed for the screen, image scaling has found uses outside of special effects. The results of scaling between faces have proved of interest to psychologists. The amount of information the human brain infers from the subtleties of facial features is enormous. The ability to bring about fluid changes and to somehow combine the salient features of faces has opened up new fields of research in the areas of face recognition and human perception. More sophisticated scaling algorithms have also been adopted by computer vision researchers looking at the broader field of object recognition. In these methods, a computer will attempt to scale an observed object into a selection of objects stored in a database. Using a measure of

how ‘easy’ it was to image scale from the original to each database object, the computer will then decide which of the database objects it is seeing.

We limit ourselves here to study of the basic image scaling algorithms world of cinema. Initially we examine the methodology behind various image scaling techniques, with particular focus on applying the mesh warping technique to both still images and video footage.

## II. METHODS:

### Gaussian scale mixtures in the wavelet domain

For each coefficient in the pyramid representation, we consider a neighborhood of coefficients, referring to the center coefficient as the reference coefficient of the neighborhood. The neighborhood may include coefficients from other subbands (i.e., corresponding to basis functions at nearby scales and orientations), as well as from the same subband. For this work we have used a  $3 \times 3$  neighborhood around the reference coefficient, plus the parent coefficient (same orientation and position, next coarser scale), whenever it exists. Due to sampling differences at different scales, the coarser subband must be resampled at double rate in both dimensions for obtaining the parent of each coefficient of the subband.

We use a Gaussian scale mixture (GSM) to model the coefficients within each local neighborhood with reference coefficients belonging to every given subband. A random vector  $x$  is a Gaussian scale mixture[3] if it can be expressed as the product of a zero-mean Gaussian vector  $u$  and an independent positive scalar random variable  $p_z$ :

$$x = \sqrt{z}u. \quad (1)$$

The variable  $z$  is the multiplier. Vector  $x$  is an infinite mixture of Gaussian vectors, whose density is determined by the covariance matrix  $C_{uu}$  of  $u$  and the mixing density  $p^z(z)$ :

$$p_x(X) = \int p(X|z) p_z(z) dz \\ = \int \frac{\exp(-X^T(zC_{uu})^{-1} X/2)}{(2\pi)^{N/2} |zC_{uu}|^{1/2}} p_z(z) dz, \quad (2)$$

where  $N$  is the dimensionality of  $x$  and  $u$  (the size of the neighborhood). Without loss of generality one can assume  $E\{z\} = 1$ , which implies  $C_{uu} = C_{xx}$ .

GSM densities are symmetric and zero-mean, and they have



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leptokurtotic marginal densities (i.e., heavier tails than a Gaussian). Another key property of the GSM model is that the density of  $x$  is Gaussian when conditioned on  $z$ . Also, the normalized vector  $x/\sqrt{z}$  is Gaussian. They also present interesting joint statistics: the variance of a vector element conditioned on a neighbor scales roughly linearly with the square of the neighbor's value. These marginal and joint statistics of GSM distributions are qualitatively similar to those of neighbor coefficients responding to natural images in multi-scale and multi-orientation representation.

### Bayes least squares estimator

For each neighborhood, we estimate  $x_c$ , the reference coefficient, from  $y$ , the set of noisy coefficients. The Bayes least squares (BLS) estimate is:

$$\begin{aligned} \mathbb{E}\{x_c|y\} &= \int x_c p(x_c|y) dx_c \\ &= \int \int_0^\infty x_c p(x_c|y, z) p(z|y) dz dx_c \\ &= \int_0^\infty p(z|y) \mathbb{E}\{x_c|y, z\} dz. \end{aligned} \quad (7)$$

Thus, the solution is an average of the least squares estimate of  $x_c$  when conditioned on  $z$  (local Wiener solution), weighted by the posterior density of the multiplier,  $p(z|y)$ . This integral can be computed numerically for each neighborhood of coefficients with a few uniform samples in  $\log z$ . We now describe each of these components.

### Local Wiener estimate

The local linear estimate for the full neighborhood is the Wiener solution:

$$\mathbb{E}\{x|y, z\} = z C_{xx'} (z C_{x'x'} + C_{ww})^{-1} y, \quad (8)$$

where  $C_{xx} = C_{xx'} H^T$  is the  $M \times N$  cross-covariance matrix of  $x$  and  $x' = Hx$ , the coefficients from the original image and those from its blurry version. We explain below a method for estimating this matrix.

We can simplify the dependence of this expression on  $z$  by diagonalizing the matrix  $z C_{x'x'} + C_{ww}$ . Specifically, let  $S$  be the symmetric square root of the positive definite matrix  $C_{ww}$ . (i.e.,  $C_{ww} = SS^T$ ), and  $\{Q, \Lambda\}$  the eigenvector / eigenvalue expansion of the matrix  $S^{-1} C_{x'x'} S^{-T}$ . Then:

$$\begin{aligned} z C_{x'x'} + C_{ww} &= z C_{x'x'} + SS^T \\ &= S (z S^{-1} C_{x'x'} S^{-T} + I) S^T \\ &= S Q (z \Lambda + I) Q^T S^T. \end{aligned} \quad (9)$$

This diagonalization does not depend on  $z$ , and thus only needs to be computed once for each subband. We can now simplify:

$$\begin{aligned} \mathbb{E}\{x|y, z\} &= z C_{xx'} S^{-T} Q (z \Lambda + I)^{-1} Q^T S^{-1} y \\ &= z M (z \Lambda + I)^{-1} v, \end{aligned} \quad (10)$$

### Multi-scale image scaling

In multi-scale techniques the scale-specific features need to satisfy the following criteria namely (i) Causality (ii) Edge localization and (iii) Scale calibration.

The conventional opening and closing and hence the top-hat transformation do not satisfy these while those by reconstruction does. However, filtering by reconstruction incurs large computational cost. Since in the proposed method, the image is subjected to filtering for small number of low-valued scale factors the issues stated above are not so stringent and hence the conventional opening and closing may be used instead of opening and closing by reconstruction respectively.

In the proposed work, we recombine the features extracted using multi-scale top-hat filters by assigning large weights to small features.

### Thin Plate Spline Based Image scaling

Thin-plate Spline is a conventional tool for surface interpolation over scattered data. It is an interpolation method that finds a "minimally bended" smooth surface that passes through all given points. The name "Thin Plate" comes from the fact that a TPS more or less simulates how a thin metal plate would behave if it was forced through the same control points.

Let us denote the target function values  $v_i$  at locations  $(x_i, y_i)$  in the plane, with  $i=1, 2, \dots, p$ , where  $p$  is the number of feature points. In particular, we will set  $v_i$  equal to the coordinates  $(x_i', y_i')$  in turn to obtain one continuous transformation for each coordinate. An assumption is made that the locations  $(x_i, y_i)$  are all different and are not collinear.

The TPS scaling allows an alignment of the points and the bending of the grid shows the deformation needed to bring the two sets on top of each other (b). In the case of TPS applied to coordinate transformation we actually use two splines, one for the displacement in the  $x$  direction and one for the displacement in the  $y$  direction. The two resulting transformations are combined into a single mapping.

## III. CONCLUSION

The focus of this article has been to survey various image scaling algorithms sufficient information. In doing so we have defined a few easily comparable attributes, such as visual quality of scaling, the ease with which the animator can select control pixels and the computational complexity. The Thin Plate Spline gives results, which are of comparable quality with very little effort required. The algorithm significantly works for larger number of feature lines to give the same results. In summary, we feel that the Thin Plate Spline image scaling based Image algorithm is the best choice since it produces good quality results. There are a variety of Image scaling algorithms such as Image scaling with snakes and free formed deformations, Image scaling using deformable surfaces, Image scaling using Delaunay triangulation and many others

besides. Due to lack of resources and time, we are unable to provide a comprehensive comparison of these algorithms.

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