A Method for the Stress and Fatigue Analysis of Bolted Joint Connections: together with Programmed Solution

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Abstract: Often the weakest link in integral engineering equipment, bolted joint connections require proper attention and detailed analysis at the design stage for a fail safe operation in service. The analysis is often lengthy with several variables under consideration. A step-by-step guide, together with all required equations for evaluating a typical bolted joint connection is given. A computer programmed solution in Microsoft Excel® for such analysis is shown through a worked example.

Index Terms: Bolt and nut connection, bolted joint analysis, bolt fatigue, joint stresses, bolt preload.

I. INTRODUCTION

The literature on the analysis of bolt and nut connections is vast. It is also the subject of certain Country or Engineering institutional codes – such as Engineering Sciences Data Unit Guide (ESDU), American Society of Mechanical Engineers (ASME), British Standards (BS), German VDI and others [1]-[6]. The major interest in all the standards is the question: Will the joint or connection fail in service? Failure in service here will imply a dynamic condition; as for example, effect of excitations on, a flowing oil and gas pipeline bolted flanged connection, the hinged end-cover of a pressurized heat exchanger vessel, hold-down foundation bolts of an engine, aged effect through environmental degradation and corrosion on the bolted connections of bridge frameworks or the chassis of a motor vehicle or marine vessel, street or other lamp posts, truss linkages of telecommunication masts, general equipment vibration effects resulting in bolt loosening. But, static failure is also a source for worry. Field experience shows that it is not unusual for bolts to break during tightening. What level of tightening or torque then is adequate? Can the applied tightening torque provide the required preload or initial tension to keep the joint together and avoid separation of the connection while in service?

II. ENGINEERING MATHEMATICAL RELATIONSHIPS

A. Sectional Area Relationships

Tensile Stress Area:

\[ A_t = \frac{\pi D^2}{4} \]  

Cross-sectional Area of body:

\[ A_b = \frac{\pi D^2}{4} \]  

Thread Lengths:

For L>125 mm, \( L_t = 2D + 6 \)  

For 125 mm<L<200 mm, \( L_t = 2D + 12 \)  

For L>200 mm, \( L_t = 2D + 25 \)  

Nominal body Length:

\[ L_{bn} = L - L_t \]  

Effective body Length:

\[ L_{beff} = L_{bn} + \left( \frac{B}{2} \right) \]  

Effective thread Length:

\[ L_{eff} = L_g - L_{beff} + \left( \frac{N}{2} \right) \]  

B. Stiffness Calculation Relationships

The principle upon which the stiffness is obtained in a bolt and nut connection is based on the deformation of an axially loaded bar guided by Hooke’s law. The deformation, \( \delta \), for such cases is defined by (9):

\[ \delta = \frac{FL}{AE} \]  

Hooke’s Law:

\[ k = \frac{F}{\delta} = \frac{AE}{L} \]  

Using a bolt and nut to tighten an engineering member, stretches the bolt and induces tension load in the bolt and compression in the clamped members. A bolt, from Hooke’s Elastic law consideration is viewed as a series combination of a head/shank body section, and a threaded section. The stiffness of the body and threaded sections are thus obtained separately, and the effective total bolt stiffness, calculated through a series combination.

Stiffness of body section:

\[ k_{be} = \frac{A_b E}{L_{beff}} \]
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Stiffness of threaded section:

\[ k_{te} = \frac{A_tE}{L_{teff}} \]  

(12)

Stiffness of Clamped Member:

\[ k_m = \frac{\pi E_s}{4L_p} \left\{ \frac{D_b}{2} + \left( \frac{L}{10} \right)^2 \right\} - D^2 \]  

(13)

Total Stiffness of bolt:

\[ k_t = \frac{(k_m + k_{te})}{(k_{be} + k_{te})} \]  

(14)

Total Joint Stiffness:

\[ k_r = k_t + k_m \]  

(15)

Stiffness Parameter of bolt:

\[ Y_b = \frac{k_b}{k_t} \]  

(16)

Stiffness Parameter of member:

\[ Y_m = \frac{k_m}{k_t} \]  

(17)

Resilience of bolt

\[ R_t = \frac{1}{k_b} \]  

(18)

Resilience of clamped member

\[ R_m = \frac{1}{k_m} \]  

(19)

C. Load Evaluation Relationships

Proposed Preload:

A bolt is often employed to connect two or more parts together. Preload or pretension determines the strength of the joint connection [7]. Recommended values for preload as suggested in the open literature vary. Shigley [8], [9] suggests a preload of 90 percent of the proof strength. Baumann [10] recommends a preload value of 70% - 80% of the static tensile strength. This for a safer condition is best taken as the yield strength of the bolt material. This is also backed by Brenner’s [11] argument, that a bolt torque tightened to induce a preload does not develop as high a tensile strength as one statically loaded, the reason being the additional torsion component due to the torque. Brenner [11] further states that, because of the straight line relationship between increasing tensile load and bolt stretch up to the yield point of the bolt, a bolt torque tightened within its yield strength will be capable of developing the full rated tensile strength when subjected to additional load in excess of the preload. Thus,

\[ P_L = \gamma \sigma_{sp} A_t \]  

(20)

Where, \( \gamma \) is the preload-yield factor.

Make-up Torque:

ESDU [2] provides a relation for make-up torque defined by equation (21),

\[ T = P_t \left( \frac{(D+D_p)}{2} \left( \frac{\mu_s}{2} \right) + \frac{p}{2\pi} \left( \frac{Dh_f}{2} \right) \cos \theta \right) \]  

(21)

Where, \( p \) is the metric thread pitch.

III. ANALYSIS DURING TIGHTENING

A first step to analysis is the evaluation of the static state of the bolt during tightening to answer such questions as: Will the applied preload shear the bolt? In plain language, will the bolt break? Will the applied static preload sustain the connection under dynamic condition, i.e., operation in service? What is the static working stress? The reliability of the joint connection as judged through an index of reliability or factor of safety under static condition when considered alongside the bolt material type selected? The following relations help answer these questions:

Tension Load or tensile stress induced in bolt body (with tightening torque):

\[ \sigma_{bt} = \frac{P_L}{A_b} \]  

(23)

Torsional Shear Stress:

\[ \tau = \frac{16T}{\pi D^3} \]  

(24)

Principal Stress or Working Stress in bolt

\[ \sigma_1 = \frac{\sigma_{bt}}{2} + \sqrt{\left( \frac{\sigma_{bt}}{2} \right)^2 + \tau^2} \]  

(25)

IV. ANALYSIS AFTER TIGHTENING – EVALUATION IN SERVICE

Tension load in bolt:
$F_p = (Y_p F_{ex}) + P_L$ \hspace{1cm} (26)

Maximum tensile stress in bolt in service

$\sigma_b = \frac{M y_{NA}}{I_E}$ \hspace{1cm} (35)

$\sigma_{bs} = \frac{F_p}{A_b}$ \hspace{1cm} (27)

Compressive load induced in member

$F_{e1} = (Y_m F_{ex}) - P_L$ \hspace{1cm} (28)

Minimum Preload to prevent loss in compression

$P_{L, min} = Y_m F_{ex}$ \hspace{1cm} (29)

Joint Adequacy for Fatigue loading:

Equivalent Stress in Shank section

$\sigma_{eqv} = \sigma_{bs} + K_s \left( \frac{\sigma_{yp}}{\sigma_y} \right) \sigma_b$ \hspace{1cm} (30)

Equivalent Stress in threaded section

$\sigma_{eqv} = \sigma_{bs} + K_t \left( \frac{\sigma_{yp}}{\sigma_y} \right) \sigma_b$ \hspace{1cm} (31)

Stress concentration factors, $K_s$, $K_t$, are available from Peterson [14]. Area reduction or transitions such as bolt head fillet or chamfer, the start of the first thread and fillets in the plane of the nut are points of stress concentration [9]. Endurance or fatigue limit is derived experimentally. When a value is not available, the practice is to make an estimate as a percentage of the yield strength [15].

Factor of safety for bending/fatigue:

Fatigue assessment is based on the Soderberg criterion and the calculation of a safety factor or reliability index based on fatigue is:

$n_f = \frac{\sigma_{yp}}{\sigma_{eqv}}$ \hspace{1cm} (32)

Angular Turn of the Nut:

$C_L = \frac{360 P_t}{k_s P}$ \hspace{1cm} (33)

Moment of Inertia through bolt axis:

$I_y = \left( \frac{\pi D^3}{4} \right) y_{NA}$ \hspace{1cm} (34)

Maximum Bending Stress:

\begin{align*}
&\text{Fig.(1): - Basic Mathematical model used to formulate relationships} \\
&\text{NA = Neutral Axis} \\
&\text{N'A' = Transfer Axis} \\
&\text{Note:} \\
&y_{NA} = \frac{D_t}{2} + \frac{(D_w - D_s)}{2}
\end{align*}

V. NUMERICAL EXAMPLE

If the bolt in Fig. (1) is an ISO M24 bolt subjected to combined bending and tensile separating forces:

a) Conduct a detailed stress evaluation analysis of the joint connection?

The bolt and Nut details are:

Bolt Diameter, $D_s=24.84$ mm

Height of Bolt Head, $B=15.9$ mm

Nut Thickness, $N=18.84$ mm

Nut Width across flat, $S=36$ mm

The basic bolt length, $L=120$ mm

Clamping length = 100 mm

$E_{bolt}=E_{member}=2.07 \times 1010$ Pa

Take, $\sigma_y=2.75 \times 108$ Pa

$\sigma_{yp}=6.90 \times 108$ Pa

Separating Load, $F_{ext} = 10,000$ N

Bending Moment, $M=130$ Nm
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VI. CONCLUSION

Figs. (2) and (3) and (4) shows that changing the preload value through altering the preload-to-yield factor, significantly influences the degree of reliability of the joint. The question of an adequate preload is thus best answered by conducting a what-if type analysis with the program. This is also in line with Aaronson’s [1] design check for static load condition:

\[
\text{Preload} \leq 70\% \text{ of load at yield point}
\]

Design check for Fatigue loading condition:

Bolt working load or equivalent stress ≤ load at yield point.

Nomenclature

- \( A_t \): Tensile Stress Area, \( \text{mm}^2 \)
- \( A_b \): Cross-Sectional area of body, \( \text{mm}^2 \)
- \( B \): Height of Head, mm
- \( C_l \): Angular Turn of the Nut, mm
- \( D \): Nominal Bolt Diameter, mm
- \( D_s \): Body Diameter, mm
- \( E \): Elastic Modulus, \( \text{N/mm}^2 \)
- \( F_{\text{ext}} \): External Load, N
- \( I_E \): Equivalent Moment of Area, \( \text{mm}^4 \)
- \( k_b \): Stiffness of Bolt, \( \text{N/mm} \)
- \( k_m \): Stiffness of Clamped member, \( \text{N/mm} \)
- \( k_t \): Stress Concentration factor for Thread
- \( K_t \): Stress Concentration factor for Shank section
- \( L_g \): Clamping or Grip Length, mm
- \( M \): Bending Moment, Nm
- \( n_f \): Factor of Safety
- \( N \): Nut Thickness, mm
- \( p \): Pitch of threads, mm
- \( P_t \): Nominal Preload, N
- \( P_{\text{Lmin}} \): Minimum Preload, N
- \( R_b \): Resilience of bolt, \( \text{mm/N} \)
AUTHORS PROFILE

Tonye K. Jack is a Registered Engineer, and ASME member. He worked on plant maintenance and rotating equipment in the Chemical Fertilizer industry, and on gas turbines in the oil and gas industry. He has Bachelors degree in Mechanical Engineering from the University of Nigeria, and Masters Degrees in Engineering Management from the University of Port Harcourt, and in Rotating Machines Design from Cranfield University in England. He was the Managing Engineer of a UK engineering software company, Alignographics. He is currently a University Teacher in Port Harcourt, Rivers State, Nigeria, teaching undergraduate classes in mechanical engineering. His research interests are on rotating equipment engineering, maintenance, engineering management, engineering computer programs, and applied mechanics.

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