Analysis of Thrust Coefficient in a Rocket Motor

P. Bose, K.M Pandey

Abstract*: *In motors of artillery rockets and anti tank missiles solid propellant is used to provide high thrusts for short period of time. On fixing of propellant composition and its grain geometry nozzle design becomes the controlling factors for optimum performance of rocket. Thrust coefficient is one of the most important parameters for its performance. It is the thrust per unit chamber pressure and throat area. It is a dimensionless multiplication factor and signifies the degree to which the thrust is amplified by the nozzle. It is a function of gas property i.e. specific heat ratio of the gas and other thermodynamic parameters. It is also a function of nozzle geometry i.e. expansion ratio and pressure ratio. It is highest when the nozzle expands the gases exactly down to ambient pressure at the exit plane. However, thrust coefficient is independent of chamber pressure. In this paper thrust coefficient is analysed as a function of expansion ratio at three different values of specific heat ratio. It is observed that flow separation typically occurs when the ratio of exit pressure to atmospheric pressure is less than 0.25 to 0.35 and thus kept less than 0.40. Thrust coefficient losses are due to divergence of the flow at the nozzle exit, skin friction losses, two-phase flow and also propellant performance. These are minimized by developing proper propellant and designing suitable nozzle. However, the losses cannot be brought down to zero. The paper analyses the various parameters that affect the thrust coefficient and brought out methods to improve the performance of solid rocket motor.

Index Terms*: *Flow separation, Nozzle throat area, Propellant, Rocket motor, Thrust coefficient.

I. INTRODUCTION

A solid propellant rocket motor is a convenient package for providing high thrusts for short period of time and thus suitable for military use as artillery rockets and anti tank missiles. Thrust is obtained as the sum of rate of change of momentum flux (F_{mom}) and pressure force (F_{pr}) acting on the rocket in the direction of motion. F_{mom} is directly proportional to the mass rate of flow of exhaust gas ($\dot{m}_p = \frac{dm_p}{dt}$) due to propellant burning at the exit of nozzle.

Once grain design and propellant composition is fixed for a rocket motor, the nozzle and its design becomes the controlling factors for its performance. For design of nozzle, chamber pressure, thrust and total impulse are the controlling parameters. There is however a number of other design and performance parameters, coefficients and correction factors which are equally important for nozzle design. One of these is thrust coefficient.

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II. THRUST COEFFICIENT AND ITS SIGNIFICANCE

The degree to which the thrust is amplified by the nozzle is quantified by the Thrust Coefficient (C_F)**,** and is defined in terms of the chamber pressure and throat area. It is the thrust (F) per unit chamber pressure (p_c) and throat area (A_*). The significance of thrust coefficient is that it determines the amplification of the thrust due to the gas expansion in the nozzle as compared to that would be exerted if chamber pressure acted over the throat area only without the gas expansion in the nozzle. It is a dimensionless multiplication factor and used for determining the thrust with known value of combustion pressure and the nozzle throat area. Mathematically it is expressed as under :-

$$C_F = \frac{F}{p_c A_*}$$

Thrust coefficient can also be expressed in terms of characteristic velocity (c^*) and specific impulse (I_s). As we know, $c^* = \frac{p_c A_*}{\frac{dm_p}{dt}}$ and $I_s = \frac{F}{\frac{dm_p}{dt}}$ which on mathematical manipulation provides the relation between c^* and I_s with C_F as given below :-

$$C_F = \frac{I_s}{c^*}$$

In a solid propellant rocket motor it is very difficult to measure $\frac{dm_p}{dt}$. Hence the performance parameters are defined in terms of integrated values wherein it is convenient to measure operating pressure and burning time. The parameters are as under :-

$$c^* = \frac{\int p_c A_* dt}{m_p}$$

$$I_s = \frac{\int F dt}{m_p}$$

$$C_F = \frac{\int F dt}{\int p_c A_* dt}$$

A well designed nozzle will deliver a C_F of about 1.5 under steady-state conditions. Ideal C_F for the same nozzle would be around 1.65. A large fraction of the loss is due to two-phase flow inefficiencies. C_F is a function of gas property i.e. specific heat ratio (γ) of the gas, nozzle area ratio i.e.

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expansion ratio (A_e/A_*) and pressure ratio across the nozzle i.e. the ratio of throat pressure (p_t) to exit pressure (p_e). However, it is independent of chamber pressure and depends on the nozzle geometry. Thrust coefficient (C_F) and thrust (F) have peak value when exit pressure (p_e) is equal to ambient pressure (p_a), expansion of gas under this condition is called ideal expansion. This peak value is referred as optimum thrust coefficient and important criteria for nozzle design.

III. ANAALYSIS OF THRUST COEFFICIENT

In Fig 1 and 2, the C_F has been plotted as function of expansion area ratio, A_e/A_*) for a given p_a/p_c (1 x 10^{-3} and 5 x 10^{-3}) at three different values of specific heat ratio (γ = 1.15, 1.2 and 1.3). The data have been determined using ideal rocket motor analysis. Similar relations can be plotted for

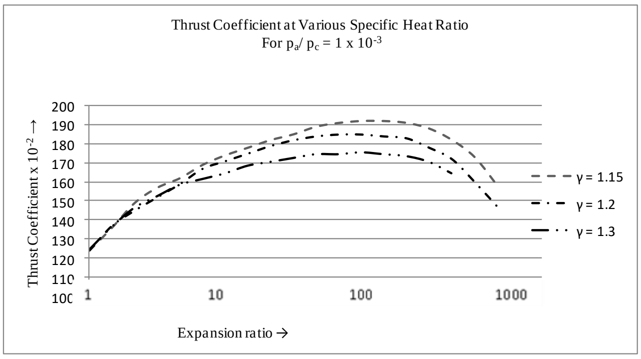

Figure 1

other p_a/p_c (zero, 25 x 10^{-3} and 50 x 10^{-3}). It is observed that C_F increases with the increase of expansion ratio till it reaches a maximum value. This is the C_F at ideal expansion point (p_a/p_c = 1) of the gas for a given γ. It decreases sharply on further increase of expansion ratio and culminates at one point as at the beginning. However, peak C_F decreases with the increase of p_a/p_c. All C_F plots show decreasing trend with the increase of γ.

Thrust coefficient depends on the geometry of nozzle and thermodynamic parameters of the evolved gas due to propellant burning. These are as follows :-

1) C_F is inversely proportional to throat area (A_*) of the nozzle for a given chamber pressure (p_c). Hence with other parameters remaining constant C_F decreases with the increase of A_*.
2) C_F increases with the decrease of pressure ratio [the ratio of chamber pressure (p_c) to exit pressure (p_e)]. However, C_F is considered once pressure ratio is fixed. Hence it has no direct impact on C_F.
3) It decreases with the increase in specific heat ratio (γ) of the gas due to decrease of C_v. With the increase in specific heat ratio exit velocity of gas decreases causing reduction of momentum flux (F_{mom}).
4) It increases with the increase of expansion area ratio [the ratio of nozzle exit area (A_e) to throat area (A_*)] due to increase of pressure force (F_{pr}). However there is an optimum value of expansion area ratio for a given value of the pressure ratio where the thrust coefficient if maximum. Beyond this expansion area ratio thrust coefficient drops sharply.

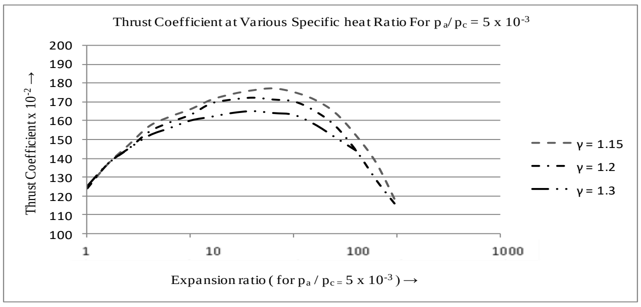

Figure 2

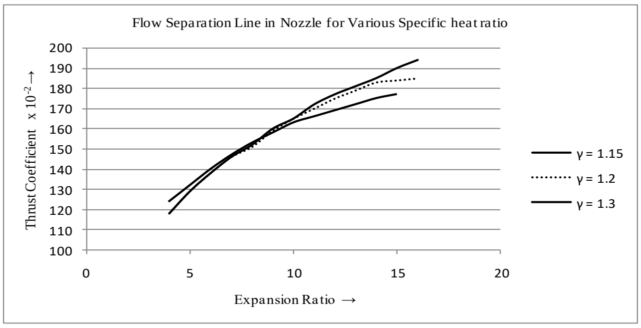

Figure 3

It is observed that flow separation of burnt gas occurs in the nozzle after C_F crosses its peak for a particular p_a/p_c and γ. This point is termed as separation point and separation continues for all expansion ratios to the right of the separation point. By joining the separation points for various p_a/p_c but at specific γ separation line is plotted. Fig 3 shows flow separation lines for various γ. It also exhibits decreasing trend with the increase of γ. Separation typically occurs when p_e/p_a is less than 0.25 to 0.35. To be on the safer side, it is taken $p_e/p_a < 0.40$.

In the table below values of C_F is tabulated against various values of pressure ratio (p_c/p_e) for a given γ.

	C_F for various values of specific heat ratio (γ)		
p_c/p_e	γ =1.15	γ = 1.2	γ = 1.3
10	1.274	1.268	1.261
20	1.422	1.408	1.388
100	1.680	1.645	1.589
200	1.766	1.720	1.650
1000	1.927	1.858	1.754
2000	1.983	1.904	1.786
10000	2.091	1.990	1.843

Table1. Thrust Coefficient (C_F) for Various Values of Specific Heat Ratio (γ)

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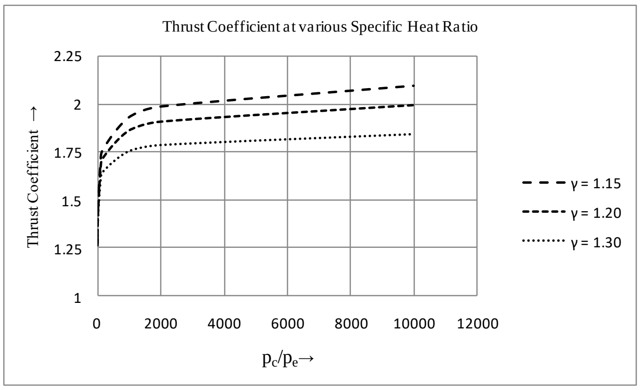

Figure 4

IV. LOSSES AFFECTING THRUST COEFFICIENT

The losses [5] which affect the thrust coefficient include divergence of the flow at exit from the nozzle, skin friction losses within the nozzle, and two-phase flow. In addition the thrust may be reduced because the flow area at the throat is less than the geometric area due to throat curvature and the presence of the boundary layer.

Thrust coefficient is maximum when the nozzle expands the gases exactly down to ambient pressure at the exit plane. Expansion area ratio greater than this can be used, but the pressure at the exit would fall below the ambient pressure. This would result shock separation of the flow within the nozzle, with separation point moving upstream to a point at which the static pressure in the exhaust flow is close to the ambient value. The conditions for flow separation depend on the Mach number of the flow and the ratio of the local static pressure to the ambient pressure. It is important to recognize that even for an infinite expansion into a vacuum, the thrust coefficient will only just exceed 2, while in most instances even a simple sonic orifice will provide a thrust coefficient of about 1.25. This limits the possible variation of the thrust coefficient, and in practice at the start of a design it should be fairly easy to guess the value of thrust coefficient within about 10%.

A. Divergence Loss

1) Let us consider correction to the throat area. The thickness of the boundary layer at the throat will generally be about 0.2% of the throat diameter. So it provides only a small correction. More important is the presence of the throat radius of curvature R_c, the loss in effective cross section due to throat curvature is approximately

$$1-\left(\frac{\gamma+1}{96}\right)\left(\frac{R_*}{R_c}\right)^2$$

2) If radius of curvature at the throat becomes less than the throat radius, then the loss in effective cross-section becomes considerable.

3)The divergence loss to the thrust coefficient will depend on the exit half angle θ of the nozzle. Manila[2] showed that by considering the exit flow as part of a cone, the contribution of the divergence loss to the nozzle thrust coefficient efficiency was

$$\eta_d = \frac{1+\cos\theta}{2}$$

B. Skin Friction Drag Loss

It is normally 1% of the thrust. It increases for long nozzle designs with subsonic or supersonic blast pipes. A very approximate estimate for the contribution of skin friction to thrust coefficient efficiency is

$$\eta_t = 1 - 0.004\frac{L}{D_*}\frac{A_*}{A_c}$$

C. Loss due to propellant impurities

With aluminized propellant a large fraction of exhaust consists of liquid or solid particles which accelerate with expanding gas but do not contribute to the thermodynamic process. For about 16% of aluminum in the propellant formulation, about one quarter of the exhaust mass flow in Al_2O_3, and if the velocity lag of the particles on exit is about 30% (that is, their velocity is 0.7 that of the gas, then the loss of specific impulse is about 7.5%. However much of this loss occurs upstream of the throat and becomes included in corrections to c^*. If we rely on measured value of c^*, then the correction to thrust coefficient will be much less (about 2%).

D. Loss due to poor workmanship

It is intended the nozzle to have well designed with continuous interior contour. However it is difficult to achieve during manufacturing in micro structure level. There are discontinuities in the nozzle contour gradient – the most common case being to include a parallel throat section with an abrupt transition to a conical expansion. With this kind of irregularities, the assumption that the flow is two dimensional becomes invalid. Instead the flow turns the corner through a Prandtl – Meyer expansion with a sudden drop of pressure. In such a case the loss of thrust coefficient is around 4 - 5%.

E. Additional Losses

These losses are due to the rate of change of gas temperature as it moves down the nozzle and thus destabilizes the gas composition from its chemical equilibrium. However in solid propellant rocket motor these losses are small and more predominant in liquid propellant motor.

V. REDUCTION OF LOSSES

Skin friction drag loss is primarily due long length of nozzle exit. This loss can be minimized by incorporating design optimization with bell shape contour. Such nozzles – referred to as 'Rao' nozzles after their inventor [6] – operate by turning the flow back toward the axis of the nozzle at the maximum rate possible without forming core shocks at the nozzle axis. The design of such nozzles is generally done using CFD software like FLUENT etc.

Other losses[7] attributable to propellant impurities, poor workmanship and losses due to the rate of change of gas temperature can be minimized by selecting proper propellant, employing better manufacturing process. However, it may not be practically possible to bring the losses to zero. It may be minimized to minimum by using advanced scientific method and state of the art engineering and technology.

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VI. CONCLUSION

Thrust coefficient is one of the most important parameters for nozzle design in the case of solid propellant rocket motors. As brought out above, it is a function of exhaust gas property and nozzle geometry. It increases with the increase of expansion ratio till it reaches a maximum value. This is the thrust coefficient at ideal expansion point of the gas for a given γ. It decreases sharply on further increase of expansion ratio. It exhibits decreasing trend with the increase of γ. It is affected due to divergence loss, skin friction loss, flow separation, loss of energy due presence of impurities in propellant, poor workmanship during manufacturing etc. Flow separation of tge gas in the nozzle occurs when p_e/p_a is less than 0.25 to 0.35. It is avoided by ensuring $p_e/p_a < 0.40$ while designing the nozzle. By taking care of design factors and propellant as brought out, thrust coefficient of rocket motor can be greatly improved.

VII. ACKNOLEDGEMENT

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REFERENCES

1. Edwin D Brown, Model rocket engine performance, *Technical notes of ESTES Industries*, Colorado, USA, 2.
2. Nakka R A, Grain design and performance evaluation, *Solid Propellant Rocket Motor – Design and Testing* by Richard Allan, pp. 1-3, (1984).
3. HMSO, Thrust coefficient analysis, *Text Book of Ballistics and Gunnery*, vol. 2, 1986, pp. 79-81.
4. Yahya S M, Relation of thrust coefficient with specific heat ratio, *Fundamentals of Compressible Flow with aircraft and Rocket Propulsion*, 2006, pp. 234-337.
5. AMCP Pamphlet, Sources of energy, *Elements of Armament Engineering*, vol. 1, 1979, pp. 321-322.
6. Sutton G P and Biblarz O, Rocket nozzle, *Rocket propulsion elements*, 1982, pp. 156-157.
7. Royal Military College of Science, Minimising losses in rocket nozzle, *The handbook of Artillery weapons*, 1984, pp. 298-299.
8. www.lr.tudelft.nl/live/pagina.jsp
9. www.456fis.org/THE BLACKBIRD AND NASA
10. www.nakka-rocketry.net
11. www.rimworld.com
12. www.aerospaceweb.org

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Dr. K.M.Pandey did his Ph.D in Mechanical Engineering in 1994 from IIT Kanpur. He has published and presented more than 200 papers in International & National Conferences and Journals. Currently he is working as Professor of the Mechanical Engineering Department, National Institute of Technology, Silchar, Assam, India. Department, National Institute of Technology, Silchar, Assam, India. Pandey has also taken the responsibility of Dean, Faculty Welfare at NIT Silchar since November 2011.

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