

Signal Code Modulation for Broadband Wireless Systems

Anubhuti Khare, Manish Saxena, Pravin J Chaudhari

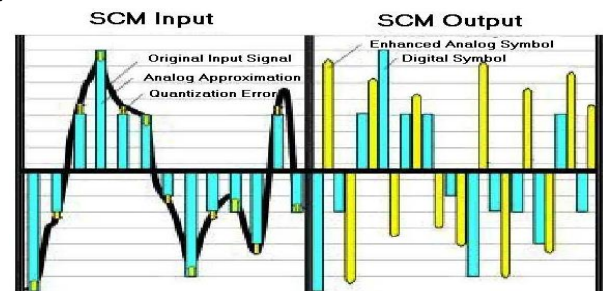
Abstract— This paper seeks to present ways to eliminate the inherent quantization noise component in digital communications, instead of conventionally making it minimal. It deals with a new concept of signaling called the Signal Code Modulation (SCM) Technique. The primary analog signal is represented by: a sample which is quantized and encoded digitally, and an analog component, which is a function of the quantization component of the digital sample. The advantages of such a system are two sided offering advantages of both analog and digital signaling. The presence of the analog residual allows for the system performance to improve when excess channel SNR is available. The digital component provides increased SNR and makes it possible for coding to be employed to achieve near error-free transmission

Index Terms—SCM, Hybrid Modulation, Quantized residual amplification.

I. INTRODUCTION

Let us consider the transmission of an analog signal over a band-limited channel. This could be possible by two conventional techniques: analog transmission, and digital transmission, of which the latter uses sampling and quantization principles. Analog Modulation techniques such as Frequency and Phase Modulations provide significant noise immunity as known and provide SNR improvement proportional to the square root of modulation index, and are thus able to trade off bandwidth for SNR. However, the SNR improvement provided by these techniques is much lower than the ideal performance as shown by the Shannon's capacity theorem [1]. On the other hand, Digital techniques of transmission can utilize error correction codes that provide performance close to theoretical prediction. However, the major disadvantage of digital transmission techniques is the inherent quantization error introduced, which is imminent all the while the signal is relayed. This error causes distortion in the original signal being relayed and cannot be later

recovered by any means possible. If we quantize the sampled signal using QAM or any other method, using a fixed number of bits, a fixed digital distortion is introduced in the developmental stage itself. This distortion is present regardless of the transmission quality of the channel being used. Thus the original signal can be considered to be permanently impaired. Communications systems are normally constructed for SNR much higher than the minimum that is required, so as to leave a margin for fading and other effects which might occasionally reduce the SNR [2]. So, it is essential to design a communications system where the output SNR increases as the channel SNR increases. While, as already stated, this technique is not feasible through digital modulation, it is an inherent property in analog modulation. Here, we introduce the concept of Signal Code Modulation (SCM) which utilizes both the analog, as well as, digital modulation techniques. The primary analog input signal is sampled at the appropriate rate and quantized. The digital samples are denoted by symbols D. The resulting D symbols are then transmitted using digital transmission techniques (like QAM) optimized for that channel. Those D symbols represent N bits per analog input sample. The quantization residual, which is not left behind, is transmitted over the noisy channel as an analog symbol A, corresponding to the digital symbol D, as shown in the figure 1.



To produce the quantization error A, the quantized data is converted back into analog form and subtracted from the original analog input signal. This symbol A, for noise immunity, is amplified by a gain of $2N$ (or any proportional factor that will optimize the voltage swing of the signal with that of the channel). The SCM receiver performs the opposite operation by combining the D symbol and its corresponding residual. This would not bring about significant improvement if transmitted over a noisy channel as noise could vary the symbol A and cause bit errors in the D symbols.

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However, the 2N amplitude gain of the analog components provides a noise immunity of 22N to boost the SNR and provide a near ideal scheme for error-free transmission.

II. THE SCM TECHNIQUE: AN ANALYTICAL APPROACH

Suppose we are given a band limited signal of bandwidth B Hz, which needs to be transmitted over a channel of bandwidth B_C with Gaussian noise of spectral density N₀ watts per Hz. Let the transmitter have an average power of P watts. We consider that the signal is sampled at the Nyquist rate of 2B samples per second, to produce a sampled signal x(n). Next, let the signal be quantized to produce a discrete amplitude signal of M=2^b levels. Where b is the no. of bits per sample of the digital symbol D, which is to be encoded. More explicitly, let the values of the 2^b levels be, q₁ q₂ q₃ q₄..... q_M which are distributed over the range α[-1, +1], where α is the proportionality factor determined relative to the signal. Given a sample x(n) we find the nearest level q_{i(n)}. Here, q_{i(n)} is the digital symbol and x_a(n)= x(n)-q_{i(n)} is the analog representation. The exact representation of the analog signal is given by x(n)=q_{i(n)}+x_a(n). We can accomplish the transmission of this information over the noisy channel by dividing it into two channels: one for analog information and another for digital information. The analog channel bandwidth is B_a=β_aB, and the digital channel bandwidth being B_d=β_dB, where B_a+B_d=B_c, the channel bandwidth. Let β=B_c/B, be the bandwidth expansion factor, i.e. the ratio of the bandwidth of the channel to the bandwidth of the signal. Similarly, the variables β_a and β_d are the ratios of B_a/B and B_d/B. Here we will assume that β_a=1 so that β_d=β-1. The total power is also divided amongst the two channels with fraction p_a for the analog channel and fraction p_d for the digital one, so that p_a+p_d=1. The SNR of the channels is first conveniently defined where no bandwidth expansion is used:

$$\gamma = \frac{P}{BN_0} \tag{1}$$

The SNR of the analog channel is given by:

$$SNR_a = \frac{P_a P}{B_a N_0} = \frac{P_a}{\beta_a} \gamma \tag{2}$$

And the SNR of the digital channel is given by:

$$SNR_d = \frac{P_d P}{B_d N_0} = \frac{P_d}{\beta_d} \gamma \tag{3}$$

Of special interest is the case where the signal power is divided in proportion to bandwidth. This is the case where the analog and digital channels have the same spectral density of the transmitted signal. Inferring that in this case:

$$SNR_a = SNR_d = \gamma / \beta \tag{4}$$

The objective of the communication system is to transmit the signal x(n) as accurately as possible. In other words, we want to design the system so as to maximize, the output SNR of the demodulated signal x̂(n), where the output SNR is:

$$SNR_0 = \frac{E\{(\hat{x}(n) - x(n))^2\}}{E\{x^2(n)\}} \tag{5}$$

In the following, we calculate the output SNR as the function of the channel SNR and the bandwidth expansion factor and plot the corresponding graph.

III. MAXIMUM OUTPUT SNR

Let us consider the best possible SNR that can be obtained by bandwidth expansion, when we wish to transmit a signal of bandwidth B through a Gaussian channel of bandwidth βB.

It can be derived using Shannon’s capacity theorem that the formula for capacity of a Gaussian Channel is given by:

$$C = \beta B \log_2(1 + \gamma / \beta) \tag{6}$$

While at the demodulator end, we have,

$$C = B \log_2(1 + SNR_0) \tag{7}$$

The two capacities must be equal since both contain the same information. Equating the two yields

$$SNR_0 = (1 + \frac{\gamma}{\beta})^\beta - 1 \tag{8}$$

The figure 2 depicts the Output SNR vs. γ for different bandwidth expansion factors.

IV. PERFORMANCE COMPARISON

SCM offers near ideal communications performance. To show this is true, let us consider the role of a communications link designer who has a noisy transmission channel of bandwidth B and limited SNR. Let us choose a digital link as a first and best choice. Here, the analog samples are converted to digital with a resolution of b bits per sample.

According to Shannon’s principle of the capacity of a noisy transmission channel, by using an ideal error correction coding technique the information can be passed error free at a bit rate equal to channel capacity, given by equation (7).

If the analog signal is sampled at a rate of R samples per second. Then, the number of bits per symbol cannot exceed b= C/R. Thus M=2^b is fixed and quantization error is unavoidable. The designer may consider analog modulation, such as FM, which is known to increase the output SNR. FM accomplishes this advantage at the expense of bandwidth increase. FM is inferior to PCM at the minimum channel SNR. This is because FM suffers from a threshold phenomenon where the performance decreases drastically with channel SNR.[3]



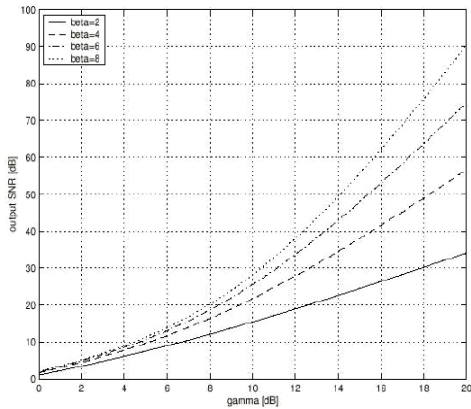


Figure 2. Output SNR versus γ (channel SNR) for different bandwidth expansion factors

A. The Ideal SCM

Now let us consider the SCM technique with the mixed analog/digital link: Assume for the moment that the digital symbols are transmitted error free. Note: the analog symbol $x_a(n)$ produced by the SCM process described above, has a smaller variance than the original symbol $x(n)$. Consider the case when $x(n)$ is a uniformly distributed random variable. Assuming that $x(n) \in [-\gamma, +\gamma]$. As there are $2B$ symbols/sec and C bits/sec, we have $b=C/2B$ bits per symbol. Now the analog sample in the range $[-\gamma, +\gamma]$ is not

transmitted in full, instead it is divided into $M=2^b$ equal segments and only one segment consists of the analog information. This segment is magnified to the range $[-\gamma, +\gamma]$ and transmitted with PAM. The b bits associated with it are transmitted through the digital channel and recovered. The receiver in turn will take the analog signal, shrink it by 2^b times and translate it to its original level. Analytically, the residual $x_a(n) \in 2^{-b}[-\gamma, +\gamma]$. That is to say, the amplitude of $x_a(n)$ is exactly $g_a = 2^b$ times smaller than that of the original signal. This means that we can amplify the analog residual by g_a to give it the same size and power of the original signal. Here, we can define g_a as,

$$g_a^2 = \frac{E\{x^2(n)\}}{E\{x_a^2(n)\}} \quad (9)$$

The amplified analog signal $g_a x_a(n)$ will be transmitted through the analog channel which has a signal to noise ratio SNR_a . At the receiver, the original signal will be reconstructed by,

$$\hat{x}(n) = \hat{q}_{i(n)} + \hat{x}_a(n) \quad (10)$$

where, $\hat{x}(n)$ is the estimated analog symbol, and $\hat{q}_{i(n)}$ is the estimated digital symbol, both of which are assumed to be equal to the transmitted symbol. Because the transmitted analog symbol was $g_a x_a(n)$, the received analog symbol will need to be divided by g_a to produce $\hat{x}_a(n)$. This will reduce channel noise by g_a and consequently improve the SNR experienced by the analog symbol by the factor of g_a^2 .

We could conclude that in general,

$$SNR_0 = g_a^2 SNR_a \quad (11)$$

And therefore

$$SNR_0 = g_a^2 \frac{P_a \gamma}{\beta_a} \quad (12)$$

The analog gain of an uniformly distributed input is $g_a=2^b$, and therefore

$$SNR_0 = 2^{2b} \frac{P_a \gamma}{\beta_a} \quad (13)$$

Here we can note that, different distributions of the analog signal lead to different analog gain factors. The gain is largest for a uniformly distributed input and becomes smaller as the distribution approaches a Gaussian distribution. The above result assumes that the digital symbol is transmitted error-free. However, this is not totally true. This raises questions, how many bits in the channel can be transmitted error free? Which leads us to the Shannon derived capacity of a digital channel, which is

$$C_d = \beta_d B \log_2 \left(1 + \frac{P_d \gamma}{\beta} \right) \quad (14)$$

If we assume that the signal $x(t)$ was sampled at the Nyquist rate of $2B$ samples per second, then the number of bits per sample will be

$$b = \frac{C_d}{2B} = \frac{\beta_d}{2} \log_2 \left(1 + \frac{P_d \gamma}{\beta_d} \right) \quad (15)$$

Note: The b is a continuous function of the SNR of the channel, γ . If we want the number of bits to be integer, then

$$b = \left[\frac{\beta_d}{2} \log_2 \left(1 + \frac{P_d \gamma}{\beta_d} \right) \right] \quad (16)$$

where $[x]$ denotes the integer part of x .

Finally,

$$SNR_0 = 2^{2 \left[\frac{\beta_d}{2} \log_2 \left(1 + \frac{P_d \gamma}{\beta_d} \right) \right]} \frac{P_a \gamma}{\beta_a} \quad (17)$$

If we assume, that the transit power is allocated in proportion to the bandwidth, we have

$$SNR_0 = 2^{2 \left[\frac{\beta_d}{2} \log_2 \left(1 + \frac{P_d \gamma}{\beta_d} \right) \right]} \frac{\gamma}{\beta} \quad (18)$$



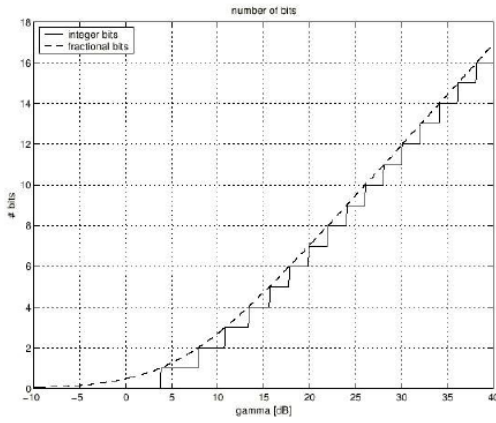


Figure 3. No. of bits in a digital symbol as a function of channel SNR γ , for an integer no. of bits (solid) and fractional no. of bits (dashed).

$$\beta = 4$$

If we allow for fractional bits, the above equation becomes,

$$SNR_0 = \left(1 + \frac{\gamma}{\beta}\right)^{\beta_d} \frac{\gamma}{\beta} \quad (19)$$

Figures 3 and 4 illustrate the performance of the ideal SCM as given by equation (18). Figure 3 shows how the bits vary with channel SNR, and figure 4 shows how the output SNR varies with the channel SNR γ . Here we can note that the ideal SCM provides performance, which is quite close to the bound.

The close match between the SCM performance and the SNR_0 bound can be proved analytically.

For large γ , the SNR_0 bound is

$$SNR_0 = \left(1 + \frac{\gamma}{\beta}\right)^{\beta} - 1 \approx \left(\frac{\gamma}{\beta}\right)^{\beta} \quad (20)$$

Similarly, for large γ , the SNR_0 of ideal SCM (19) becomes

$$SNR_0 \approx \left(\frac{\gamma}{\beta}\right)^{\beta_d} \frac{\gamma}{\beta} = \left(\frac{\gamma}{\beta}\right)^{\beta_d+1} \quad (21)$$

Equation (20) and (21) are the same since, $\beta_d+1 = \beta$ For small γ , the SNR_0 bound gives $SNR_0 \approx \gamma$, while for ideal SCM we have $SNR_0 \approx \gamma / \beta$.

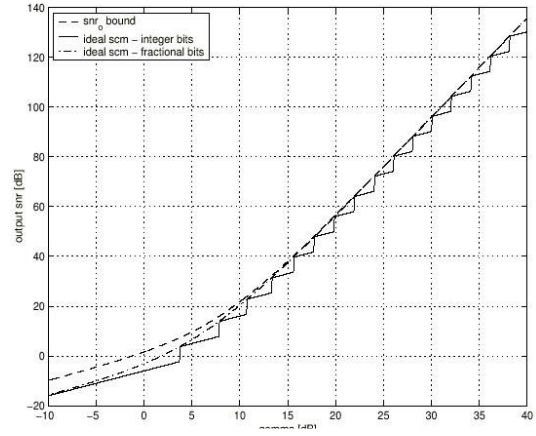


Figure 4. The output SNR_0 as a function of channel SNR γ , for the SNR_0 bound (dash), the ideal SCM with integer bits (solid) and the ideal SCM with fractional no. of bits (dash-dot). $\beta = 4$.

1. Ideal SCM with Power Optimization

The performance of the SCM technique can be further improved by adjusting the power allocation between the analog and digital channels so as to maximize the output SNR. Figures 5 and 6 depict the performance of the ideal SCM for allocation of optimal power, computed by adjusting p_d in equation (17) so as to maximize SNR_0 .

We see from figure 5 that the ideal SCM characteristics approach that of the Shannon bound for both high and low input SNR values. Figure 6 shows the number of bits used, and the power allocation to the digital channel. These results can be verified analytically by computing the optimum value of p_d for the case of fractional bits, and evaluating the corresponding SNR_0 .

Recalling equation (17) and modifying it by removing the “[]”, we get

$$SNR_0 = \left(1 + \frac{p_d \gamma}{\beta_d}\right)^{\beta_d} \gamma (1 - p_d) \quad (22)$$

Differentiating SNR_0 with respect to p_d , setting the derivative to zero to find the maximum p_d , we get for $\gamma \geq 1$

$$p_d = \frac{\gamma - 1}{\gamma} \frac{\beta_d}{\beta_d + 1} \quad (23)$$

While for $\gamma < 1$, we have $p_d = 0$. Inserting this into equation (22) we get, after some straightforward manipulations, and using the relation $\beta = \beta_d + 1$, that for $\gamma \geq 1$

$$SNR_0 = \left(1 + \frac{\gamma - 1}{\beta}\right)^{\beta} \quad (24)$$

It is straightforward to check that for $\gamma \gg 1$ and for $\gamma \ll 1$ the output SNR of the ideal SCM approaches the Shannon bound.

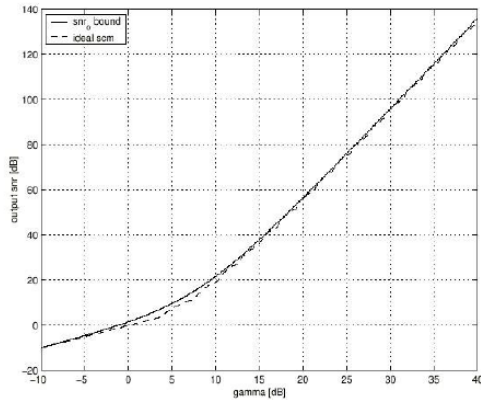


Figure 5. The output SNR as a function of channel SNR γ , for the ideal case (solid), and the ideal SCM with the integer bits, with power optimization. ($\beta = 4$)

2. Ideal SCM with fixed no. of bits

When the number of bits transmitted through the channel are fixed, say $b=b_0$. In this case, the analog gain g_a will be fixed, and the output SNR_0 will be, $SNR_0 = 2^{2b_0} \gamma / \beta$, provided that the digital transmission is error free. This will

$$\gamma_0 = \beta (2^{\frac{2b_0}{\beta_d}} - 1) \quad (25)$$

For SNR values below the threshold there will be a high probability of error and we assume for simplicity that the output SNR will drop to zero. Though this is an extreme consideration, in reality, the degradation in SNR_0 will be more gradual.

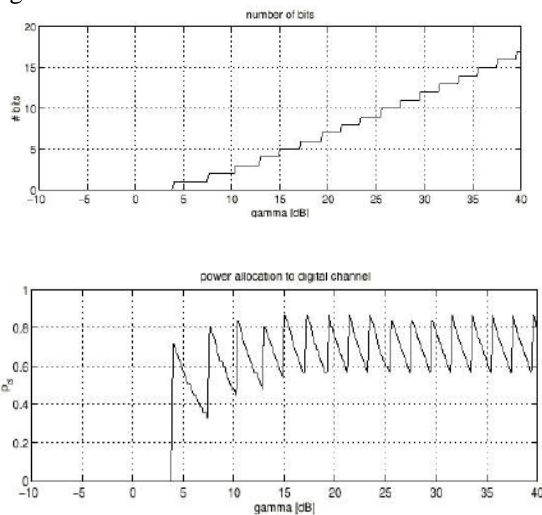


Figure 6. The number of bits and the optimal power p_d as function of γ for the case shown in figure 5. $\beta=4$

Figure 7 depicts the output SNR as a function of channel SNR with a fixed number of bits. As expected, the output SNR increases linearly with γ , and there is a constant SNR gain equal to the analog gain g_a^2 .

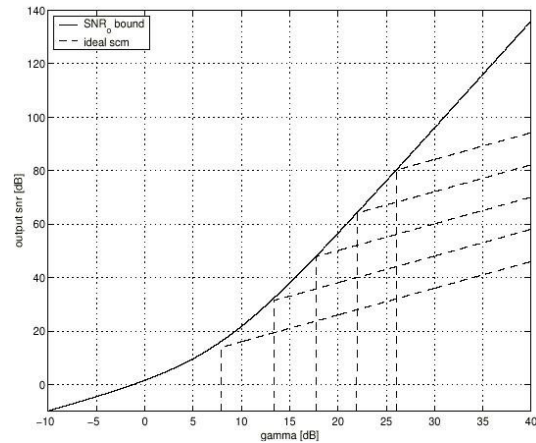


Figure 7. The output SNR as function of channel SNR γ for the SNR_0 bound (solid), and the ideal SCM with fixed number bits and $\beta = 4$. From bottom to top $b_0 = 2, 6, 8, 10$.

V. PROSPECTIVE APPLICATIONS

1 Broadband Wireless Transmission

An SCM-based communications link is basically a transparent, band-limited analog pipe with near-ideal performance in noisy channels. Every analog signal could potentially use SCM because it can outperform other existing modulation schemes. However, SCM has a compelling advantage for digital communications applications as well.

For example, SCM can pass digital information by acting as a repeater of a digital channel. This application provides a wireless extension of cable modem digital information. As illustrated in Figure 8, a cable modem termination system (CMTS) transmits a 42 Mb/s 256-QAM signal in a 6 MHz cable channel shared among the cable modems located at the subscribers' premises. The return upstream path from the cable modems is a 10 Mb/s 16-SQAM signal in a 3.2 MHz cable channel. The signals are carried by a combination of fiber and coax referred to as a hybrid fiber/coax (HFC) network.

The fiber delivers a large amount of bandwidth over long distances with strong noise immunity. Coax cables distribute the signal between the fiber and each subscriber. To reach a station located beyond the reach of the existing HFC network, the cable operator installs an SCM-based point-to-multipoint wireless access system at any point on the HFC network that has line-of-sight to the unreachable station. All customers located at a particular site share the SCM radio located at that site. The subscribers simply use low-cost cable modems that connect to the SCM radio via a shared coax cable. The wireless subscribers can even share the same cable channels with purely wired subscribers because

the wireless link is transparent to the cable equipment. The significance of SCM in this application is its ability to take a 256-QAM signal and transport it over a wireless link suitable only for a lower modulation scheme, such as 16-QAM. SCM provides significant additional noise immunity, as is depicted in Figure 2 because it uses bandwidth expansion to improve the destination SNR.



There is a non-SCM alternative: the 256-QAM signal could first be demodulated back to the original data bits, then modulated as 16-QAM, transmitted over the wireless link, demodulated at the destination, and finally remodulated using 256-QAM. This alternative would be much more

costly, given the amount of processing required. It would also add significant latency to the information transported because an efficient channel must perform the error correction of the original signal before transmitting it over the wireless link. Furthermore, because SCM provides a transparent link that is not sensitive to protocol evolution or variations, it is more future-proof and versatile than specific digital standards.

2. Superior Digital Audio Recording and Playback

A new-generation audio CD could include a digital track identical to and compatible with the existing CD tracks, and in addition, have an analog track to provide the enhanced quantization error. Such an analog track would provide audio performance that depends on the quality of the recording and of the disc player. The most discriminating audio enthusiasts could use the more sophisticated player for true analog reproduction, while the less discriminating users would enjoy the low-cost CD technology in its current format.

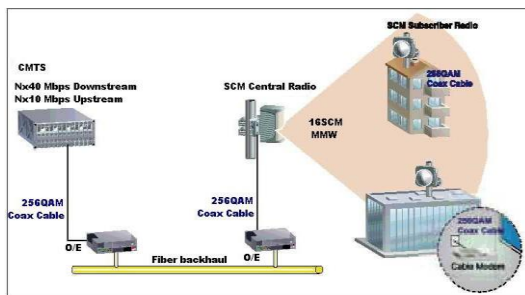


Figure 8. Communication using the SCM technique could increase efficiency and reliability while reducing interface and processing costs

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