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Fuzzy System Approximation based Adaptive Sliding Mode Control for Nonlinear System

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***Abstract:** In this paper, an adaptive sliding mode control utilizing a fuzzy system approximation is introduced. The fuzzy system is used to approximate the unknown function of an uncertain nonlinear system. The robustness of the system is ensured by the sliding mode control, while the adaptive fuzzy system improves real-time performance. To approximate unknown nonlinearities, a set of fuzzy rules is formulated whose parameters are adjusted in real-time by an adaptive algorithm. The chattering problem of sliding mode control is satisfactorily resolved, and stable operation is assured.*

***Keywords:** Sliding Mode Control, Fuzzy Logic Control; Nonlinear system; Adaptive Control; Fuzzy System Approximation.*

I. INTRODUCTION

Control theory using fuzzy logic has significantly expanded the use of controllers to manage complicated, nonlinear systems. Fuzzy logic control (FLC) has several advantages over traditional techniques, such as the ability to include human experience, expert knowledge, a flexible model-free approach, and more [1][15][16][17][18]. Fuzzy controllers are intended to function in situations where there is a great deal of uncertainty or unknown variance in the characteristics and structures of the plants [6].

Adaptive fuzzy control techniques have advanced significantly since the fuzzy system universal approximation theorem [4, 9] was proposed. They are successfully applied in many different fields, such as system modelling, signal processing, pattern recognition, system control [2, 3], etc. As a result, sophisticated controllers and complex plant representations have been created using intelligent control techniques [5, 10]. Maintaining consistent system performance in the face of these uncertainties is the general aim of adaptive control. Improved performance is attained because the adaptive fuzzy controller can adapt to its changing surroundings. The adaptive law can assist in understanding the dynamics of the plant while it is operating, and modelling is not necessary.

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To manage nonlinear systems with uncertainties, one of the strongest and most effective control strategies is sliding mode control (SMC) [8]. Attractive features include robustness to disturbance, insensitivity to matched uncertainty, and reduced order-compensated dynamics. As long as the boundaries of these disturbances are understood, it is a powerful control method that can be applied to nonlinear systems [12]. Nevertheless, sliding mode control has chattering in real-world applications and requires knowledge of the upper bound of model uncertainties as well as external disturbances [11, 14][19].

Therefore, adaptive fuzzy sliding mode control offers a great control solution for managing nonlinear and uncertain systems [7]. Hence, the design of adaptive fuzzy sliding mode controllers for nonlinear system control has been the subject of extensive research [13].

In this work, the design of adaptive sliding mode control utilising a fuzzy system approximation is introduced. The fuzzy system is used to approximate the unknown function of uncertain nonlinear system. The sliding control ensures robustness, and the adaptive fuzzy system increases the system's real-time performance.

The organization of this paper is as follows: The problem formulation is introduced in Section II. The design of a fuzzy system approximation-based adaptive sliding mode controller and stability analysis are presented in Section III. In Section IV, the simulation of a second-order nonlinear system is presented to validate the given control law. A conclusion is drawn in Section V.

II. PROBLEM FORMULATION

Let us consider a second order dynamical system as

$$\ddot{\Phi} = g(\Phi, \dot{\Phi}) + \tau + \tau_d \tag{1}$$

where Φ is angular position, $\dot{\Phi}$ is angular speed, τ is control input and τ_d is disturbance which is bounded by $|\tau_d| \leq D$, $D > 0$.

Rewrite the above equation as

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = g(z) + \tau + \tau_d \tag{2}$$

where $g(z) = g(z_1, z_2) = g(\Phi, \dot{\Phi})$ is unknown uncertainty.

Let us consider the desired angular position as z_d.

Then the error is,

$$e = z_1 - z_d \tag{3}$$

The sliding surface is designed as $s = \dot{e} + \lambda e$ where $\lambda > 0$

Then

$$\dot{s} = \ddot{e} + \lambda \dot{e} = -\ddot{z}_d + g(z) + \tau + \lambda \dot{e} + \tau_d \tag{4}$$

Now for unknown $g(z)$, we will approximate it by fuzzy approximation algorithms.

III. CONTROLLER DESIGN

Applying universal approximation theorem [4] for fuzzy system, let us design fuzzy system $\hat{g}(z|\Phi)$ to approximate $g(z)$.
For inputs z_1 and z_2 define five fuzzy sets as $B_1^{l_i}$ and $B_2^{l_i}$ respectively where $l_i = 1,2,\dots.5$, $i = 1,2$. And construct $\prod_{i=1}^{n} \rho_i = \rho_1 \times \rho_2 = 25$ fuzzy rules. The 1st and 25th fuzzy rules are given as :

$$F^1: \ if\ z_1\ is\ B_1^1\ \text{and}\ z_2\ is\ B_2^1\ then\ \hat{g}\ is\ O^1$$
$$F^{25}: \ if\ z_1\ is\ B_1^5\ \text{and}\ z_2\ is\ B_2^5\ then\ \hat{g}\ is\ O^{25}$$

Where $O^{l_1 l_2}$ is the fuzzy set of $\hat{g}$. Then based on fuzzy inference, the fuzzy system's output is,

$$\hat{g}(z|\Phi_g) = \frac{\sum_{l_1=1}^{5}\sum_{l_2=1}^{5} x_g^{l_1 l_2}\left(\prod_{i=1}^{2}\mu_{B_i^{l_i}}(z_i)\right)}{\sum_{l_1=1}^{5}\sum_{l_2=1}^{5}\left(\prod_{i=1}^{2}\mu_{B_i^{l_i}}(z_i)\right)} \qquad (5)$$

where $\mu_{B_i^{l_i}}(z_i)$ is the membership function of z_i , $x_g^{l_1 l_2}$ is a free parameter and $\Phi_g = [x_g^1 \dots\dots\dots. x_g^{25}]^T$ is parameter vector.
Equation (5) can be rewritten, based on the concept of fuzzy basis vector[4] as:

$$\hat{g}(z|\Phi_g) = \hat{\Phi}_g^T \psi(z) \qquad (6)$$

Where $z = [z_1 \ \ z_2]^T$, $\psi(z)$ is fuzzy basis vector with $\rho_1 \times \rho_2 = 25$ elements, and its $l_1 l_2$ th element is,

$$\psi_{l_1 l_2}(z) = \frac{\prod_{i=1}^{2}\mu_{B_i^{l_i}}(z_i)}{\sum_{l_1=1}^{5}\sum_{l_2=1}^{5}\left(\prod_{i=1}^{2}\mu_{B_i^{l_i}}(z_i)\right)} \qquad (7)$$

The membership functions can be selected from experience.

Now consider the optimum design parameter as

$$\Phi_g^* = \arg\ min\left[sup\left|\hat{g}(z|\Phi_g) - g(z)\right|\right] \qquad (8)$$

Where, $z \in R^2$ and $\Phi_g \in S_g$ where S_g is set of Φ_g .

Then,

$$g(z) = \Phi_g^{*T}\psi(z) + \varepsilon \qquad (9)$$

where ε is the approximation error and $\varepsilon \le \varepsilon_u$. Now,

$$g(z) - \hat{g}(z) = \Phi_g^{*T}\psi(z) + \varepsilon - \hat{\Phi}_g^T\psi(z) = -\tilde{\Phi}_g^T\psi(z) + \varepsilon$$
(10)

Now let us define the lyapunov function as,

$$L = \frac{1}{2}s^2 + \frac{1}{2\theta}\tilde{\Phi}_g^T\tilde{\Phi}_g \qquad (11)$$

Where, $\theta > 0$ and $\tilde{\Phi}_g = \hat{\Phi}_g - \Phi_g^*$, then $\dot{\tilde{\Phi}}_g = \dot{\hat{\Phi}}_g$

$$\dot{L} = s\dot{s} + \frac{1}{\theta}\tilde{\Phi}_g^T\dot{\tilde{\Phi}}_g$$
$$= s(\lambda\dot{e} + g(z) + \tau - \ddot{z}_d) + \frac{1}{\theta}\tilde{\Phi}_g^T\dot{\tilde{\Phi}}_g$$

Design control law as,

$$\tau = -\hat{g}(z) + \ddot{z}_d - \lambda\dot{e} - ksgn(s) \qquad (12)$$

Choose $k \ge \varepsilon_u + D$.

Now,

$$\dot{L} = s(g(z) - \hat{g}(z) - ksgn(s)) + \frac{1}{\theta}\tilde{\Phi}_g^T\dot{\tilde{\Phi}}_g$$

$$\dot{L} = s(-\tilde{\Phi}_g^T\psi(z) + \varepsilon - ksgn(s)) + \frac{1}{\theta}\tilde{\Phi}_g^T\dot{\tilde{\Phi}}_g$$

$$\dot{L} = \varepsilon s - k|s| + \tilde{\Phi}_g^T\left(\frac{1}{\theta}\dot{\hat{\Phi}}_g - s\psi(z)\right)$$

The adaptive law is chosen as,

$$\dot{\hat{\Phi}}_g = \theta s\psi(z) \qquad (13)$$

Then $\dot{L} = \varepsilon s - k|s| \le -k|s| \le 0$.

IV. SIMULATION RESULTS

Let us consider the plant of equation (1) as given below:

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = g(z) + \tau + \tau_d$$

Where $g(z) = 3(z_1 + z_2)$. Let the desired angular position as $z_d(t) = \sin(t)$. Choose five membership functions for z_i as:

$$\mu_{NM}(z_i) = \exp[-((z_i + \tfrac{\pi}{3})/\tfrac{\pi}{12})^2]$$
$$\mu_{NS}(z_i) = \exp[-((z_i + \tfrac{\pi}{6})/\tfrac{\pi}{12})^2]$$
$$\mu_{Z}(z_i) = \exp[-(z_i/\tfrac{\pi}{12})^2]$$
$$\mu_{PS}(z_i) = \exp[-((z_i - \tfrac{\pi}{6})/\tfrac{\pi}{12})^2]$$
$$\mu_{PM}(z_i) = \exp[-((z_i - \tfrac{\pi}{3})/\tfrac{\pi}{12})^2]$$

The initial states are [0.15 , 0], the initial value of $\hat{\Phi}$ is 0.10. The controller design parameters from equation (12) and (13) are: $\lambda = 20$, $\theta = 5000$ and $k = 0.55$. For chattering reduction saturation function is used instead of sign function with $\Delta = 0.02$.

To study the effect of disturbance the simulation results are shown with external disturbance τ_d having different amplitudes. The different amplitude disturbances are : $\tau_{d1} = 0.5\sin(t)$, $\tau_{d2} = \sin(t)$, and $\tau_{d3} = 5\sin(t)$.

The results are shown in figures (1) to (15). Results includes control input, tracking of angular position and tracking of angular speed.

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Figures (1) to (5) shows satisfactory tracking performance with negligible chattering for bounded disturbance as τ_{d1} . But for unbounded value of disturbance such as τ_{d2} and τ_{d3} slight distorted tracking is observed as shown in figures (6) to (15). The figure 16 shows the membership functions.

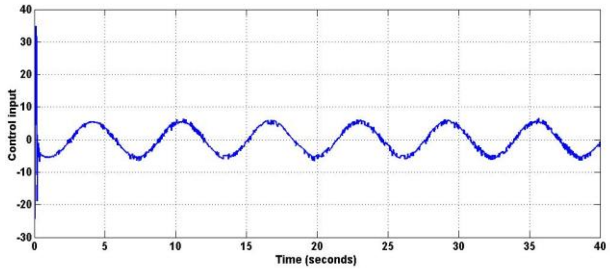

Figure 1. Control Input for τ_{d1}

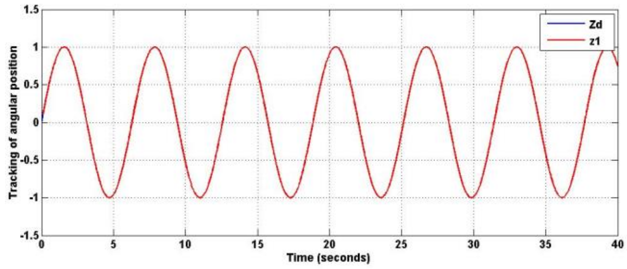

Figure 2. Tracking of Angular Position for τ_{d1}

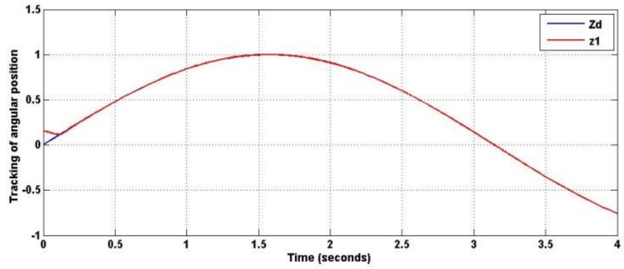

Figure 3. Tracking of Angular Position for τ_{d1} (transient)

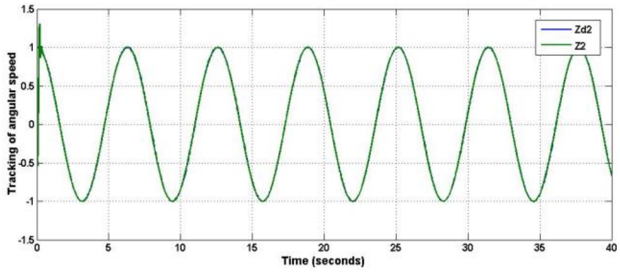

Figure 4. Tracking of Angular Speed τ_{d1}

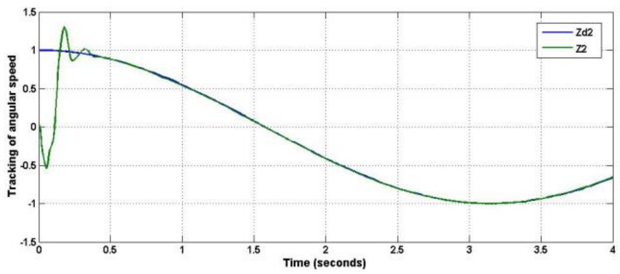

Figure 5. Tracking of Angular Speed τ_{d1}(transient)

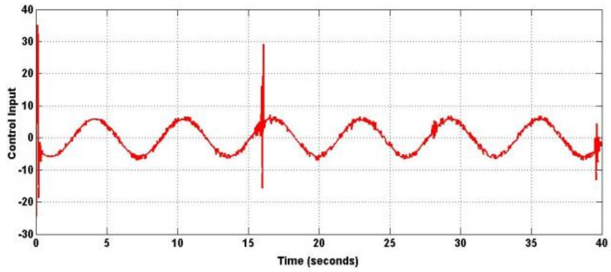

Figure 6. Control Input for τ_{d2}

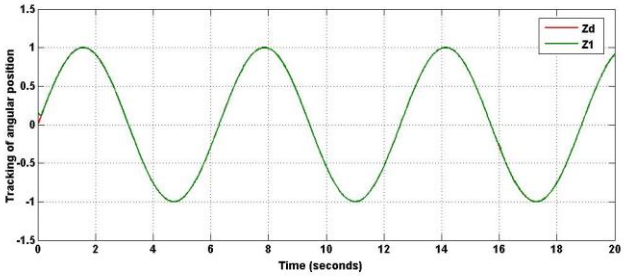

Figure 7. Tracking of Angular Position for τ_{d2}

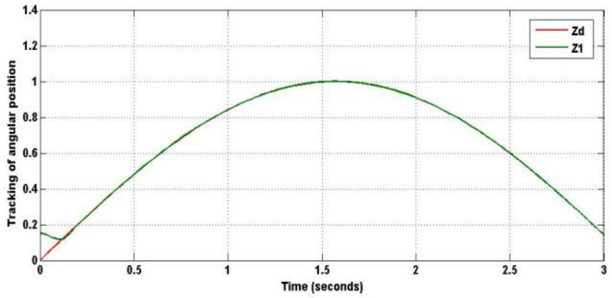

Figure 8. Tracking of Angular Position for τ_{d2}(transient)

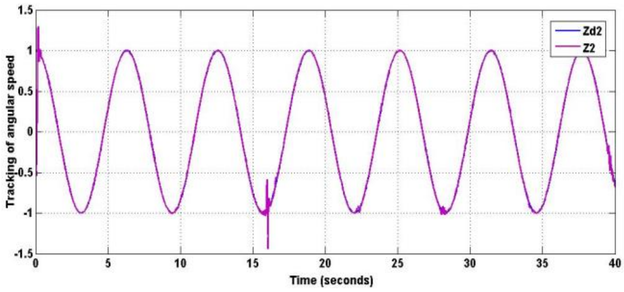

Figure 9. Tracking of Angular Speed for τ_{d2}

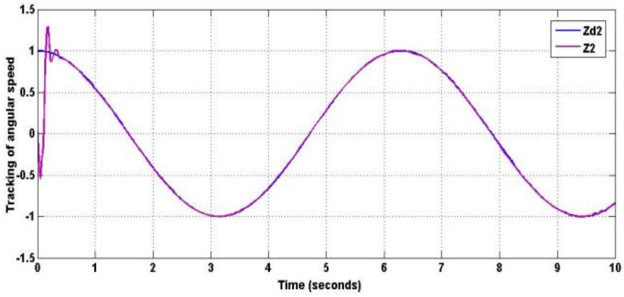

Figure 10. Tracking of Angular Speed for τ_{d2}(transient)

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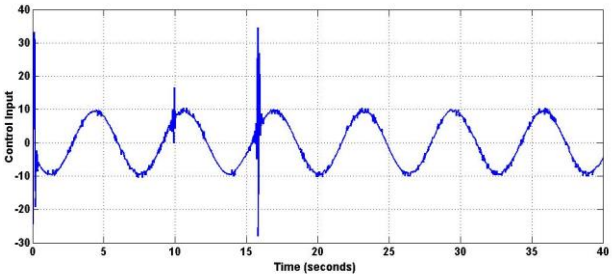

Figure 11. Control Input for τ_{d3}

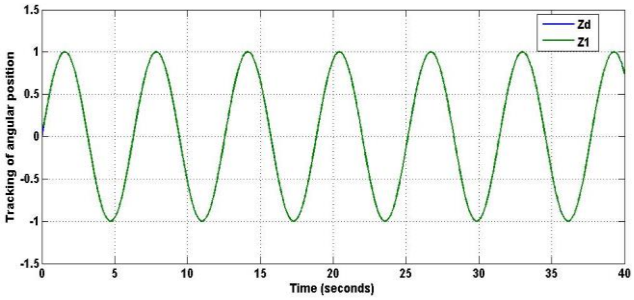

Figure 12. Tracking of Angular Position for τ_{d3}

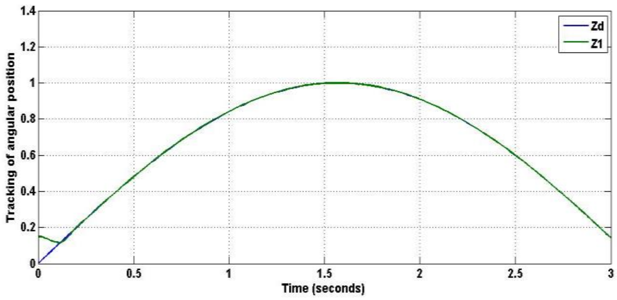

Figure 13. Tracking of Angular Position for τ_{d3}(transient)

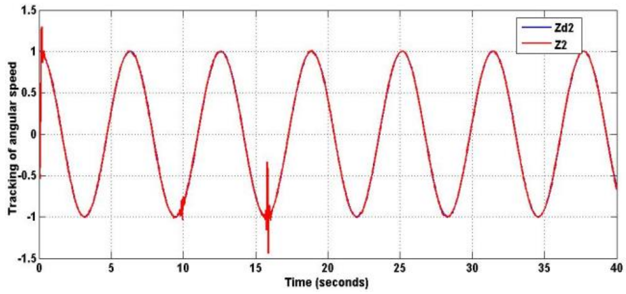

Figure 14. Tracking of Angular Speed for τ_{d3}

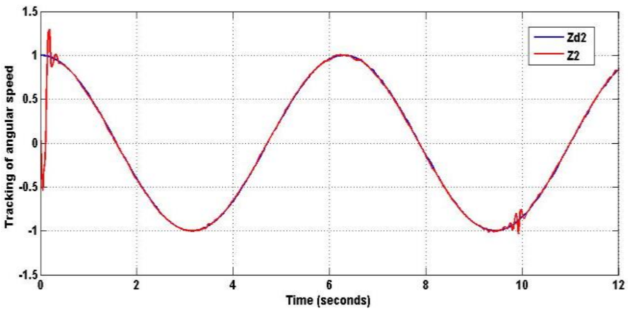

Figure 15. Tracking of Angular Speed for τ_{d3} (transient)

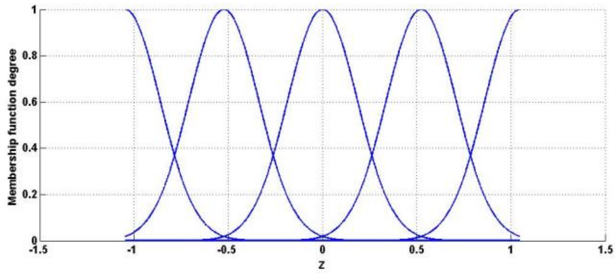

Figure 16. Membership Functions

V. CONCLUSION

This work presents an adaptive sliding mode control utilising a fuzzy system approximation for tracking control of uncertain nonlinear system. The fuzzy system is used to approximate the unknown function of the system. In order to approximate unknown nonlinearities, the fuzzy system makes use of a set of fuzzy rules, the parameters of which are continuously changed by adaptive laws. The controller ensures robust performance and the chattering action is reduced satisfactorily. The simulations results on uncertain nonlinear system for different amplitude disturbances validates the controller.

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Ethical Approval and Consent to Participate	No, the article does not require ethical approval and consent to participate with evidence.
Availability of Data and Material	Not relevant.
Authors Contributions	All authors have equal participation in this article.

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