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Square Difference Prime Labeling for Duplication of Graphs

S. Alice Pappa, G.J. Jeba Selvi Kavitha

Abstract: ***Let*** $G(V,E)$ ***be a graph with p vertices and q edges. Let*** $f: V(G) \to \{0,1,2,...,p-1\}$ ***be a bijection such that the induced function*** $f^*: E(G) \to N$ ***defined by*** $f^*_{sqdp}(uv) = |[f(u)]^2 - [f(v)]^2|$ ***for every*** $uv \in E(G)$***.If*** f^*_{sqdp} ***is injective, then*** f^*_{sqdp} ***is called square difference labeling of*** G***. A graph*** G ***which admits square difference labeling is called square difference graph. The greatest common incidence number (gcin) of a vertex v of degree*** $v > 1$ ***is defined as the greatest common divisor (g.c.d) of the labels of the incident edges on*** v***. A square difference labeling*** f ***is said to be a square difference prime labeling if for each vertex v of degree > 1 then*** $gcin(v) = 1$***.In this paper we investigate the square difference prime labelling of Petal graph and duplication of petal graph***

Keywords and Phrase: Graph Labelling, Graph Labeling, Greatest Common Incidence Number (GCIN), Square Difference Prime Labelling (SQDP), Square Difference Prime (SQDP)

I. INTRODUCTION

Here every graph is simple finite and undirected. We refer [2,3] for SQDP. Integers are allotted to the vertices or edges or both for graph labeling. Some basic notations and definitions are taken from Joseph A. Gallian [1]. Some basic hypothesis are taken from Sunoj B.S. Mathew Varkey [2,3]. Here we investigates square difference prime labelling for duplication of Petal graph

II. PRELIMINARIES

Definition 2.1. Petal graph $PT_n(G)$ is a graph obtained by appending a vertex to the first vertex of n cycle

III. SQUARE DIFFERENCE PRIME LABELING

Theorem 3.1

Petal graph $PT_n(G)$ is a SQDP graph.

Proof

Let $V = \{a,b\} \cup \{u_i/1 \le i \le n-1\}$ be the vertex set of the petal graph and $E = \{ab\} \cup \{au_1\} \cup \{u_iu_{i+1}/1 \le i \le n-2\} \cup \{u_na\}$ be the edge set of petal graph.

Here $|V(G)| = n+1$ and $|E(G)| = n+1$

Define $f: V \to \{0,1,...,p-1\}$ by

$f(a) = 0$

$f(u_i) = \{i/1 \le i \le n-1\}$ and

$f(b) = n$

Patently f is a bijection.

The induced edge labeling $f^*_{sqdp}: E(G) \to N$ is defined as follows

$f^*_{sqdp}(ab) = |[f(a)]^2 - [f(b)]^2|$

$= |0^2 - n^2|$

$= n^2$

$f^*_{sqdp}(au_1) = |[f(a)]^2 - [f(u_1)]^2|$

$= |0 - 1^2|$

$= 1$

$f^*_{sqdp}(u_iu_{(i+1)}) = \left|[f(u_i)]^2 - [f(u_{(i+1)})]^2\right|$

$= |(i)^2 - (i+1)^2|$

$= |(i)^2 - (i^2 + 2i + 1)|$

$= |i^2 - i^2 - 2i - 1|$

$= |2i+1|, 1 \le i \le n-1.$

$f^*_{sqdp}(u_{n-1}a) = |[f(u_{n-1})]^2 - [f(a)]^2|$

$= |(n-1)^2 - 0^2|$

$= |n^2 - 2n + 1|$

Patently, the edge labels are distinct.

Consequently, Petal graph $PT_n(G)$ is a square difference graph.

$\deg(a) = 3$

$gcin(a) = gcd\{f^*_{sqdp}(ab), f^*_{sqdp}(au_1), f^*_{sqdp}(u_{n-1}a)\} = gcd\{|n^2|, 1, |n^2 - 2n + 1|\} = 1$

$\deg(u_i) = 2$

$gcin(u_i) = gcd\{f^*_{sqdp}(u_iu_{i+1}), f^*_{sqdp}(u_{i-1}u_i)\} = gcd\{|2i+1|, |2i-1|\} = 1$

$\deg(u_1) = 2$

$gcin(u_1) = gcd\{f^*_{sqdp}(au_1),\ f^*_{sqdp}(u_1u_2)\} = gcd\{|1|, |2(1)+1|\} = gcd\{1,3\} = 1$

$\deg(u_{n-1}) = 2$

$gcin(u_{n-1}) = gcd\{f^*_{sqdp}(u_{n-1}a),\ f^*_{sqdp}(u_{n-2}u_{n-1})\} = gcd\{|n^2 - 2n + 1|, |2n-3|\} = 1$

Follows that $gcin$ of each vertex of degree > 1 is one.

f^*_{sqdp} is a square difference prime labeling.

Petal graph $PT_n(G)$ is a SQDP graph

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* Correspondence Author (s)

Dr. S. Alice Pappa, Department of Mathematics, Nazareth Margoschis College, Pillaiyanmanai Tuticorin-Affiliated to Manonmaniam Sundaranar University Abishekapatti, Tirunelveli (Tamil Nadu), India. E-mail: alicestephen8979@gmail.com

G.J. Jeba Selvi Kavitha, Research Scholar, Reg. No. 19232142092007, Department of Mathematics, Nazareth Margoschis College, Pillaiyanmanai Tuticorin - Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli (Tamil Nadu), India. and St. Joseph's College of Arts and Science for Women - Hosur. E-mail: jebaselvikavitha@gmail.com

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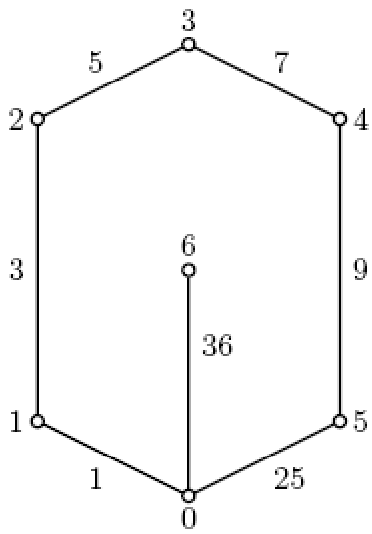


IV. DUPLICATION OF PETAL GRAPH

Theorem 4.1

Duplication of any Petal graph is SQDP graph.

Proof

Let $V = \{a, b\} \cup \{u_i / 1 \leq i \leq n-1\} \cup \{v\}$ be the vertex set of Duplication of petal graph.

Let $E = \{ab\} \cup \{au_1\} \cup \{u_i u_{i+1} / 1 \leq i \leq n-2\} \cup \{u_n a\} \cup \{av\}$ be the edge set.

Here $|V(G)| = n+2$ and $|E(G)| = n+2$

Define $f: V \to \{0,1,\dots,p-1\}$ by

$f(a) = 0$

$f(u_i) = \{i / 1 \leq i \leq n-1\}$ and

$f(b) = 0$

$f(v) = n+1$

Patently f is a bijection.

The induced edge labeling $f^*_{sqdp}: E(G) \to N$ is defined as follows

$f^*_{sqdp}(ab) = |[f(a)]^2 - [f(b)]^2|$

$= |0^2 - n^2|$

$= n^2$

$f^*_{sqdp}(au_1) = |[f(a)]^2 - [f(u_1)]^2|$

$= |0 - 1^2|$

$= 1$

$f^*_{sqdp}(u_i u_{(i+1)}) = \left|[f(u_i)]^2 - [f(u_{(i+1)})]^2\right|$

$= |(i)^2 - (i+1)^2|$

$= |(i)^2 - (i^2 + 2i + 1)|$

$= |i^2 - i^2 - 2i - 1|$

$= |2i + 1|, 1 \leq i \leq n-1.$

$f^*_{sqdp}(u_{n-1} a) = |[f(u_{n-1})]^2 - [f(a)]^2|$

$= |(n-1)^2 - 0^2|$

$= |n^2 - 2n + 1|$

$f^*_{sqdp}(av) = |[f(a)]^2 - [f(v)]^2|$

$= |0 - (n+1)^2|$

$= (n+1)^2$

$f^*_{sqdp}(u_i v) = |[f(u_i)]^2 - [f(v)]^2|$

$= |i^2 - (n+1)^2|$

$= |i^2 - n^2 - 2n - 1|$

Patently, the edge labels are distinct.

$\deg(a) = 4$

$gcin(a) = gcd\{f^*_{sqdp}(ab), f^*_{sqdp}(au_1), f^*_{sqdp}(u_{n-1}a), f^*_{sqdp}(av)\} = gcd\{|n^2|, 1, |n^2 - 2n + 1|, |(n+1)^2|\} = 1$

$\deg(u_i) = 3$

$gcin(u_i) = gcd\{f^*_{sqdp}(u_i u_{i+1}), f^*_{sqdp}(u_i v), f^*_{sqdp}(u_{i-1} u_i)\} = gcd\{|2i + 1|, |i^2 - n^2 - 2n - 1|, |2i - 1|\} = 1$

$\deg(u_1) = 3$

$gcin(a) = gcd\{f^*_{sqdp}(au_1),\ f^*_{sqdp}(u_1 u_2), f^*_{sqdp}(u_1 v)\} = gcd\{|1|, |2(1) + 1|, |1 - 2n - n^2 - 1|\} = gcd\{1, 3, |2n + n^2|\} = 1$

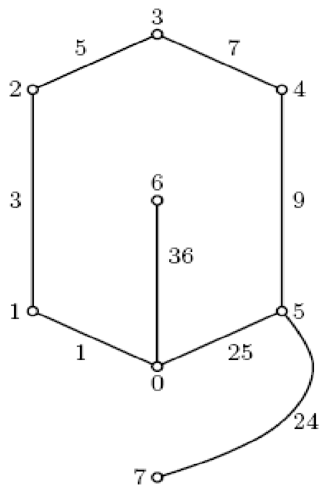
$\deg(u_{n-1}) = 3$

$gcin(u_{n-1}) = gcd\{f^*_{sqdp}(u_{n-1} a), f^*_{sqdp}(u_{n-2} u_{n-1}), f^*_{sqdp}(u_{n-1} v)\} = gcd\{|(n-1)^2|, |2n - 3|, |-4n|\} = 1$

Follows that $gcin$ of each vertex of degree > 1 is one.

f^*_{sqdp} is a square difference prime labeling.

Duplication of Petal graph $PT_n(G)$ is a SQDP graph.



V. CONCLUSIONS

Here we introduce the Petal graph. We prove petal graph is a square difference graphs. We investigate petal graph is a square difference prime graphs. Since each vertex v of degree > 1 then $gcin(v) = 1$. Duplication concept is applied to the Petal graph. We investigated petal graph is a Square difference Prime graph. We focused our investigation on some standard graph which involves square difference prime labeling.

REFERENCES

1. A Galian, A dyanamic survey of graph labeling, The electronic journal of combinatories (2019).(4529)
2. Sunoj B.S. Mathew Varkey T.K, *Square difference prime labeling for some snake graphs*, Global Journal of Pure and Applied Mathematics, ISSN: 0973-1768, Vol. 13, No. 3(2017), p.p 108-1089. (9)

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3. Sunoj B.S. Mathew Varkey T.K, *Square Difference Prime Labeling for Some path Related Graph*, International Journal of Advance Research in Science and Engineering, Vol. 13 No. 1 pp. 95-100, January-June (2017) (647)

AUTHOR'S PROFILE

Dr. S. Alice Pappa, M.Sc., M Phil., Ph.D., She is serving an Associate Professor in Department of Mathematics, Nazareth Margoschis College, Pillayanmanai at Tuticorin, Affliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012. She has an experience of 22 years. She produced a Ph.D and guiding 3 Research Scholars. She Published 9 research papers in UGC Care Listed Journal. Her Specialization is Graph Theory. She is a Member of Board of studies in Manonmaniam Sundaranar University, Tirunelveli.

G.J. Jeba Selvi Kavitha, M.Sc., M Phil., Research Scholar in Department of Mathematics, Nazareth Margoschis College, Pillayanmanai at Tuticorin, Affliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012. She Published three papers in UGC Care Listed Journal. Presented four papers in International Conferences. She is an Assistant Professor in Department of Mathematics, St. Joseph's College of Arts and Science for Women at Hosur.