International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-12 Issue-1, October 2022

Low Earth Orbiting (LEO) Satellite and Orbital Decay

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Abstract: In this article we present a simple but elegant numerical calculation to study the motion of a low earth orbiting (LEO) satellite in gravitational potential and under the influence of quadratic drag force. We solve a set of coupled differential equations to calculate the instantaneous position and velocity of the satellite and calculate the change in radial distance for different aerodynamic coefficients and altitude. We validate our numerical results with theoretical result in the high altitude where the density of air and consequently the drag force is minimum. However the LEO orbits placed in the range of 200 to 400 km faces significant effect of drag force. Satellites at an altitude of 200 km quickly decay and re-entry to earth's atmosphere occurs. We finally do a short numerical calculation with the parameters of ISS satellite at an altitude of 400 km and compare with the existing results.

Keywords: Aerodynamic Coefficient, ISS Satellite, LEO Satellite, Quadratic Drag,

I. INTRODUCTION

Satellites in all altitude regimes are affected by atmospheric drag. It is in general belief that the atmosphere at the few hundred kilometers altitude can be safely considered as a perfect vacuum. However the satellite moves very fast through the different layers of atmosphere. Particles in the atmosphere undergo collisions with the surface of the satellite. The number of collisions is magnified to a large value close to 10^{15} times (which is the average number of particles at a given height of low earth orbitals). This causes slowing down of the satellite. The effect of drag is thus cumulative and is the most important factor for orbital decay. The orbital decay is defined by the amount of orbit a satellite will lose when it completes an orbit. Thus the atmospheric drag is the major source to change the satellite position.

There are currently over 3000 active satellites orbiting the earth. The size and the altitude of a satellite depend on which purpose it has been launched. As the altitudes of satellite vary, there are three common types of orbits. Geostationary orbits (GEO) are placed 36000 km above the Earth and the period is 24 hours. The communication and weather satellites are of GEO type. The medium Earth orbit (MEO) are at altitude of 20000 km and time for one orbit is 12 hrs. Whereas low Earth orbit (LEO) satellites placed between 200 to 2000 km will face the effect of drag. Mostly the Earth observation satellites are placed in this altitude to have clear photograph of the earth. The International Space Station (ISS) are at 400 km and time for one orbit is close to 90 minutes. In past decades the number of satellites in low Earth orbit has increased and faced the particular problem with orbital decay. As the atmospheric drag is much powerful than the gravitational perturbation and the drag is proportional to the density of air which in turn depends on the altitude, ultimately causes the significant change in orbital period of LEO satellites. The satellites lose their energy, the orbit gradually contracts and essentially re-entry occurs.

In this communication we present a simple numerical solution of the set of second order coupled differential equations which govern the satellite motion under the drag force together with the Newtonian potential. The air density significantly changes with the altitude as shown in the Physical Properties of U.S. Standard Atmosphere, 1976 [1] data table. The varying air density is appropriately taken care in the calculation of drag force. We utilize the Runga Kutta method with appropriate initial condition to get the time dependent position coordinates as well as velocity components ($x(t)$, $y(t)$, $v_x(t)$ and $v_y(t)$). We do the numerical calculation for various aerodynamic coefficients and orbital heights including the specific parameters used for ISS fixed at an altitude of 400 km. We first validate the numerical results for the change in the orbital distance (Δr) with the analytical results when the air density does not change during one revolution of the satellite. We are interested to see how this simple calculation will provide to visualize the gradual decay of the circular orbit with time and ultimate fall on the earth. Thus we can estimate an average range of the altitude below which the drag plays a very crucial role for the orbital decay. If the satellite is placed in that range it will quickly fall on the earth's surface. The paper is organized as follows. In Section II, we describe the basics of the satellite motion under drag. Section III deals with our numerical results and Section IV draws our conclusion.

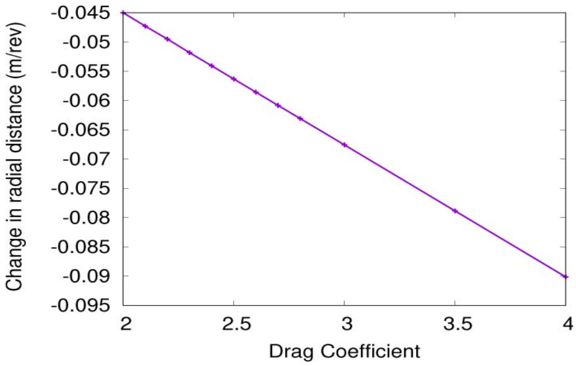

FIG. 1. The radial distance change Δr (m/rev) with various drag coefficients C_d. The satellite radial distance is 7125 km.

Manuscript received on 27 September 2022 | Revised Manuscript received on 02 October 2022 | Manuscript Accepted on 15 October 2022 | Manuscript published on 30 October 2022.

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Retrieval Number: 100.1/ijeat.A38491012122
DOI: 10.35940/ijeat.A3849.1012122
Journal Website: www.ijeat.org

Published By:
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II. SATELLITE MOTION UNDER DRAG

We consider the satellite is moving in a gravitational potential and under the influence of atmospheric drag which depends on several factors.

The atmospheric drag is taken as a product of four factors: the drag coefficient C_d, area to mass ratio of the satellite A/M_s (A is the drag area and M_s is the mass of the satellite), the atmospheric density $\rho(h)$ and the speed of the satellite with respect to the atmosphere v_s. The drag force is calculated as [2]

$$F = \frac{1}{2} C_d \left(\frac{A}{M_s}\right) \rho(h) v_s^2 \qquad (1)$$

The atmospheric density function is chosen as [3]

$$\rho(h) = \rho_0 e^{-\frac{(h-h_0)}{H}} \qquad (2)$$

Here ρ_0 is taken as the density in the reference altitude h_0. h is the orbital altitude of the satellite, H is the atmospheric scale height.

Although none of the quantities are very preciously known but we use the parameters used in conventional model calculation [4]. In general the satellites are assumed to be sphere with A/M_s close to 0.1 cm^2/gm, which is the typical value for satellite payload. The drag coefficient C_d is usually considered a constant value nearly 2.2 for the satellites in LEO. However C_d should be determined for each satellite, by calculating from the momentum and energy transferred during scattering between the surface of the satellite and the particles in the atmosphere [5]. Depending on the type of scattering the values of C_d will depend on the altitude. The value of C_d also depends on the shape of the satellite.

The equation of motion of the satellite under quadratic drag force is given by:

$$\frac{d^2\vec{r}}{dt^2} = -\frac{GM}{r^3}\vec{r} - bv\vec{v} \qquad (3)$$

which results to the set of second order coupled differential equations in the plane of the satellite

$$\frac{d^2x}{dt^2} = -\frac{GM}{(x^2+y^2)^{3/2}} - bvv_x \qquad (4)$$

$$\frac{d^2y}{dt^2} = -\frac{GM}{(x^2+y^2)^{3/2}} - bvv_y \qquad (5)$$

b is equivalent to the drag coefficient, M is the mass of the sun and the chosen value of GM is equal to 2.9979245 $\times 10^5$ (SI units). The coupled set of equations is numerically solved by Runga Kutta method to get the time dependence in x and y components of velocity and position. We calculate the change in the satellite radial distance with time.

It is to be noted that from the second order time dependent coupled differential equations (as given above), we calculate instantaneous position and velocity of the satellite. As at each instant of time the satellite changes its height, $\rho(h)$ also changes with time according to Eq. (2) and the drag force also changes with time by Eq. (1). So we are able to exhibit the most realistic situation of how the instantaneous position of the satellite is obtained under the effect of drag force.

III. NUMERICAL RESULTS

In our first calculation we consider a satellite projected in a circular orbit. The satellite mass M_s = 900 kg and projected surface area is A = 3 m^2, thus the ratio A/M_s= 3.33 $\times 10^{-3}$ m^2/Kg. The Earth radius R_E= 6378 km and the radial distance of the satellite from the center of the earth is 7125 km. Thus the orbital altitude of the satellite is 747 km, the reference density is ρ_0=3.614$\times 10^{-14}$ kg/m^3 [1]. The scale height H= 88.67 km [3]. The corresponding density at different altitude is calculated using Eq. (2).

We do the numerical computation for different starting altitude and varying drag coefficient. We calculate the change in radial distance per one revolution. If we assume that the radial change per revolution is small and density remains practically constant over one satellite period, then (Δr)
can be calculated analytically.

For the circular motion the orbital velocity is given by v_s^2=GM/r. The gravitational potential is V = -GM/r. Thus we can calculate the total energy per unit mass as

$$E = \frac{1}{2}v^2 + V = -\frac{GM}{2r} \qquad (6)$$

The change in energy per revolution can be obtained from:

$$\Delta E = -\int_0^{2\pi} \frac{1}{2} A C_d \rho(h) v_s^2 r d\theta \qquad (7)$$

If the satellite is placed in a reasonable height (nearly 700 to 800 km above the earth surface), the change in the atmospheric density is minimal and we can assume it a constant over one satellite revolution. Thus the above integration becomes simplified and the change in energy becomes

$$\Delta E = -\pi GMC_d A\rho_0 e^{-\frac{(h-h_0)}{H}} \qquad (8)$$

We can associate the change in the orbital distance with the energy loss through the equation

$$\Delta E = M_s \Delta\left(-\frac{GM}{2r}\right) = \frac{GMM_s}{2r^2}\Delta r \qquad (9)$$

Using equations (8) and (9) we find the change in radial distance as

$$\Delta r^{theo} = -\frac{2\pi C_d A r^2}{M_s}\rho_0 e^{-\frac{(h-h_0)}{H}} \qquad (10)$$

However for the satellite in LEO orbit, when the drag force is changing with the altitude as density changes, the analytical results do not hold good. To validate our numerical code we compare our numerical results with the theoretical results as shown in Table 1. We compare the change in the radial distance Δr in (m/rev) for the satellite radial distance at 7125 km for different choices of C_d= 2.0, 2.1, 2.2, 2.3 and 2.4. We observe remarkable agreement between theoretical and numerical results.

TABLE I. Comparison of the numerical results of the change in the radial distance of a satellite with theoretical results (Eq. (10)) for various drag coefficients. The satellite radial distance is $r = 7125$ km. For the choice of other parameters see the text.

drag coefficient	$\Delta r^{numerical}$(m/rev)	Δr^{theo}(m/rev)
2.0	-0.0450	-0.0451
2.1	-0.0472	-0.0474
2.2	-0.0495	-0.0496
2.3	-0.0519	-0.0519
2.4	-0.0541	-0.0541

In Fig. 1 we plot the change in the radial distance Δr (m/rev) as a function of wide range of drag coefficients. Significant change in the radial distance is observed for higher drag coefficient. The most important observation is the linear variation Δr with drag coefficient.

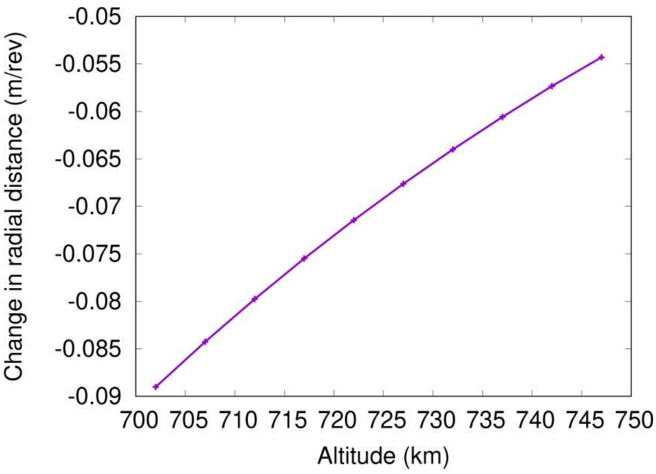

FIG. 2. The radial distance change Δr(m/rev) with different satellite altitude in the rang of 700 to 750 km with fixed drag coefficient C_d= 2.4.

In Fig. 2 we plot the change in radial distance Δr as a function of satellite altitude in the range of 700 to 750 km for fixed drag coefficient. We note that at the altitude of 747 km the value of is 5.43×10^{-2}(m/rev), whereas that at the altitude of 702 km is 8.9×10^{-2} (m/rev). This change is not significant as the air density does not vary much in this range of altitude. It is to be noted that the corresponding density 2.70×10^{-15}kg/ m^3 and 3.58×10^{-15}kg/ m^3 respectively at the altitudes mentioned above [1].

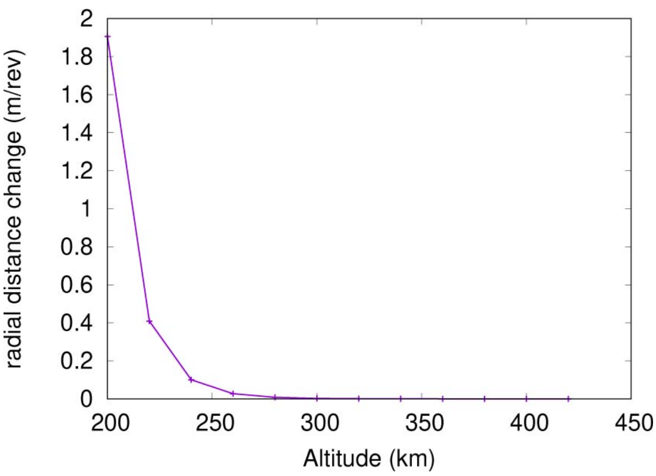

FIG. 3. Difference between the theoretical and numerically calculated "radial distance change" (δ (m/rev)) in the range of 200 to 450 km altitude above the Earth's surface with fixed drag coefficient 2.0.

We further run the code for much lower altitude where the air density changes drastically. We present results between the height of 200 km to 450 km. The density drastically changes from the order of 10^{-13}kg/ m^3 to 10^{-10}kg/ m^3 [1] as the altitude changes from 450 km to 200 km. As the air density changes significantly the theoretical formula Eq.(10) for the calculation of change in the radial distance is not valid and the orbital distance changes significantly per revolution. In Fig.3, we plot the difference between the theoretical and numerical results as δ= $\Delta r^{numerical}$ - Δr^{theo} as a function of orbital altitude. The difference is quite significant as shown in the figure.

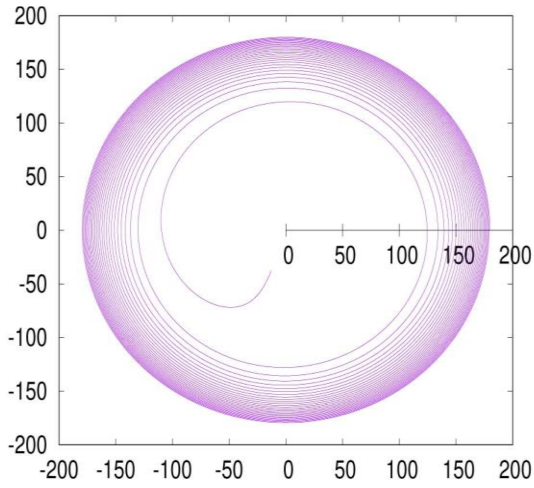

FIG. 4. Polar plot of Orbital decay for satellite altitude 180 km.

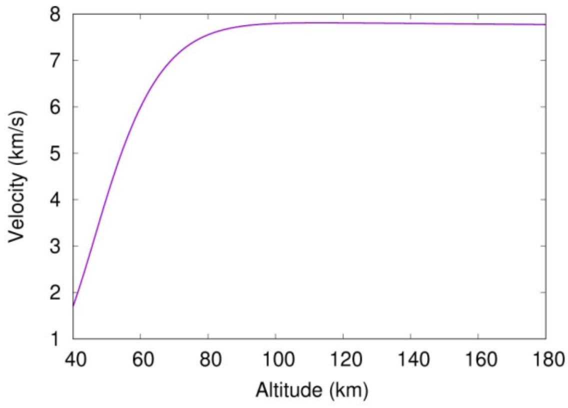

FIG. 5. Plot of speed of the satellite originally at a height of 180 km as a function of altitude.

The effect of the drag at much lower height (below 200 km) is more significant and is responsible for the instability of the orbit. The change in the radial distance is reasonably high and causes quick decay of the orbit and finally collapse close to earth surface. To exhibit this orbital decay of the satellite, we make a polar plot in Fig. 4. The satellite was projected at a height of 180 km with speed 7.17 km/s. With time the circular orbit starts to deviate and Δr increases sharply and finally tends to collapse on earth surface. The quick decay of orbital is also exhibited in Fig. 5, where we plot the speed of satellite as a function of altitude.

Retrieval Number: 100.1/ijeat.A38491012122
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We can conclude that any satellite placed in this height are unstable and re-entry to earth's atmosphere will happen typically in one or two days. However a ISS satellite generally placed close to 400 km altitude is of special interest. Here we like to find out how good is our results for the ISS satellites. According to the NASA site [6] we list the values of different parameters required to calculate the change in the radial distance. The mass M_s = 451567 Kg, projected area which experiences drag force is 1426.2 m^2, placed in the altitude of 400 km, the approximate speed is 7.7 km/s, drag coefficient C_d = 1.8. According to the NASA results, the time period of ISS is 92.68 minutes and it loses the altitude by 100 m per day. Our results are plotted in Fig.6, where we plot how the ISS satellite loses its altitude in each revolution. The graph is almost linear which exhibits that the satellite maintains almost constant time period. In 16th revolution it loses 103 m altitude and the required time is 88854 s. Thus the obtained time period is 92.57 minutes.

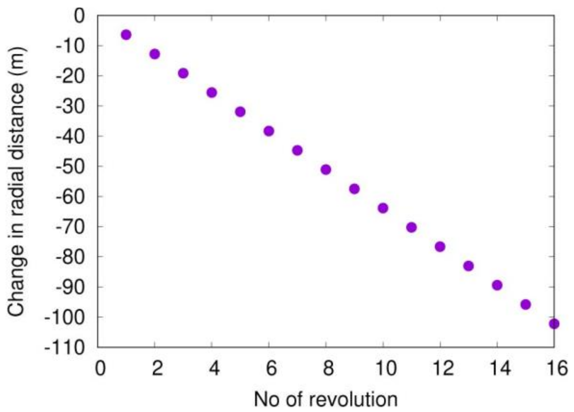

FIG. 6. Change in the radial distance of ISS satellite as a function of number of revolution. We presented up to sixteenth revolutions

IV. CONCLUSION

The present work considers the motion of satellite in a circular orbit under the influence of drag force and in the gravitational potential. We explicitly present through some numerical simulation how the effect of air density and consequently the drag force become very crucial in the determination of stable orbital of LEO satellite. As we solve the set of time dependent coupled differential equations, we predict the instantaneous position of the satellite. The calculation accurately takes care the varying air density with the altitude. So our present calculation mimics the most realistic scenario of motion of the satellite under varying drag force. When the radial distance of altitude is close to 7125 km, the change in air density is minimum and the change in radial distance will have an analytic formula. We validate our results with that formula for wide range of drag coefficient. We specially focus the case when the satellite altitude is close to 200 km and then how the satellite faces quick orbital decay and enters close to the earth surface. We also study the specific case of ISS satellite to see how this simple time dependent calculation can reproduce the time period of ISS satellite accurately. However open question remains. How good is the Newtonian potential to take into account the different perturbation during the satellite motion? In some recent article [7] and also in current theories it is predicted that r^{-2} correction to the gravitational potential is required to extract the correct physics. So redoing the numerical simulation with different forms of the correction term remains open for future study.

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