HF-Index and Y-Index of Some New Graph of Operations

S.Nagarajan, G.Kayalvizhi, G.Priyadharsini

Abstract: ***In this paper we derive HF index of some graph operations containing join, Cartesian Product, Corona Product of graphs and compute the Y index of new operations of graphs related to the join of graphs.***

Keywords: ***HF-index, Y-index, Graph Operations.***

I. INTRODUCTION

Let Ω be a simple connected graph with vertex set and edge set are V(Ω) and E(Ω) respectively. The degree of a vertex is the number of vertices in graph which are connected to an edge and it is denoted by $\delta_\Omega(v)$.

The First and Second Zagreb Indices are defined by,

$$M_1(\Omega) = \sum_{v \in V(\Omega)} \delta_\Omega(v)^2 = \sum_{uv \in E(\Omega)} [\delta_\Omega(u) + \delta_\Omega(v)]$$

$$M_2(\Omega) = \sum_{uv \in E(\Omega)} [\delta_\Omega(u)\delta_\Omega(v)]$$

The Hyper Zagreb Index are defined by ,

$$HM(\Omega) = \sum_{uv \in E(\Omega)} [\delta_\Omega(u) + \delta_\Omega(v)]^2$$

The F and HF indices are defined by,

$$F(\Omega) = \sum_{v \in V(\Omega)} \delta_\Omega(v)^3 = \sum_{uv \in E(\Omega)} [\delta_\Omega(u)^2 + \delta_\Omega(v)^2]$$

$$HF(\Omega) = \sum_{uv \in E(\Omega)} [\delta_\Omega(u)^2 + \delta_\Omega(v)^2]^2$$

The Y index are defined by,

$$Y(\Omega) = \sum_{v \in V(\Omega)} \delta_\Omega(v)^4 = \sum_{uv \in E(\Omega)} [\delta_\Omega(u)^3 + \delta_\Omega(v)^3]$$

The (a,b) Zagreb indices are defined by,[13]

$$Z_{a,b}(\Omega) = \sum_{u,v \in E(\Omega)} (\delta(u)^a\delta(v)^b + \delta(u)^b\delta(v)^a)$$

The general randic indices are defined by,[14]

$$R_\alpha(\Omega) = \sum_{u,v \in E(\Omega)} (\delta(u)^\alpha\delta(v)^\alpha)$$

Or

$$= \sum_{u,v \in E(\Omega)} (\delta(u)\delta(v))^\alpha$$

We will represent some formulas of the HF index for some graph operations such as,

The Cartesian product Ω X Ψ of graphs Ω and Ψ has the vertex set V(ΩXΨ)=V(Ω)XV(Ψ) and (a,x)(b,y) is an edge of Ω X Ψ if a=b and xyϵE(Ψ), or abϵE(Ψ) and x=y.

The join Ω + Ψ of graphs Ω and Ψ is a graph with vertex set V(Ω)UV(Ψ) and edge set E(Ω)UE(Ψ)U{uv:uϵV(Ω) and vϵV(Ψ)}.

The corona product Ω ʘ Ψ is defined as the graph obtained from Ω and Ψ by taking one copy of Ω and $|V(\Omega)|$ copies of Ψ and then joining by an edge each vertex of the i^{th} copy of Ψ is named (Ψ,i) with the i^{th} vertex of Ω.

For a graph Ω, define four operator graphs S(Ω), T(Ω), Q(Ω), R(Ω) as follows:

The graph S(Ω) is obtained by inserting an extra vertex in each edge of Ω. In other words, replacing each edge of Ω by a path of length two.

The graph R(Ω) is derived by adding a new vertex corresponding to each edge of Ω, then joining each additional vertex to the end vertices of the corresponding edge.

The graph Q(Ω) is obtained by adding a new vertex into each edge of Ω, then joining with edges those pairs of new vertices on adjacent edges of Ω, by a new edge.

The graph T(Ω) is a graph with the vertex set and two vertices are adjacent whenever they are either adjacent or incident in. In this paper, we derived some graph operations and new operations of graphs related to the join of graphs.

II. MAIN RESULTS

In this section, we discuss main results of HF index of some graph operation.

Theorem: 2.1

Let Ω and Ψ be a graphs. Then $HF(\Omega X \Psi) = 4Y(\Omega)g_2 + HF(\Psi)n_1 + 8M_1(\Omega)F_1(\Psi) + 4Y(\Psi)g_1 + HF(\Omega)n_2 + 8F_1(\Omega)M_1(\Psi)$.

Proof:

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By definition of HF index, we have

$$HF(\Omega \times \Psi) = \sum_{(r,i)(s,j)\epsilon E(\Omega X \Psi)}[\delta_{(\Omega X \Psi)}(r,i)^2 + \delta_{(\Omega X \Psi)}(s,j)^2]^2$$

$$= \sum_{r\epsilon V(\Omega)}\sum_{ij\epsilon E(\Psi)}[\delta_\Omega(r)^2+\delta_\Psi(i)^2+\delta_\Omega(r)^2+\delta_\Psi(j)^2]^2$$

$$+\sum_{i\epsilon V(\Psi)}\sum_{rs\epsilon E(\Omega)}[\delta_\Psi(i)^2+ \delta_\Omega(r)^2+ \delta_\Psi(i)^2+ \delta_\Omega(s)^2]^2$$

$$HF(\Omega \times \Psi) = Y(\Omega)g_2+HF(\Psi)n_1+8M_1(\Omega)F_1(\Psi)+4Y(\Psi)g_1+ HF(\Omega)n_2+8F_1(\Omega)M_1(\Psi).$$

which completes the proof.

Theorem: 2.2

Let Ω and Ψ be graphs. Then

$$HF(\Omega + \Psi) = HF(\Psi)+4h_1^4g_2 +8h_1^2F_1(\Psi)HF(\Omega)+HF(\Psi)+ 4(h_2^4g_1+h_1^4g_2)+ 8(h_2^2F_1(\Omega)+h_1^2F_1(\Psi))+ (M_1(\Omega)h_2+g_1h_2^3+8g_1h_2^2+ M_1(\Psi)h_1+h_1^3h_2+8h_1^2g_2).$$

Proof:

By definition of HF index, we have

$$HF(\Omega + \Psi) = \sum_{uv\epsilon E(\Omega+\Psi)}[\delta_{\Omega+\Psi}(u)^2+ \delta_{\Omega+\Psi}(v)^2]^2$$

$$= \sum_{uv\epsilon E(\Omega)} [\delta_{\Omega+\Psi}(u)^2+ \delta_{\Omega+\Psi}(v)^2]^2+ \sum_{uv\epsilon E(\Psi)} [\delta_{\Omega+\Psi}(u)^2+ \delta_{\Omega+\Psi}(v)^2]^2$$

$$+\sum_{u\epsilon V(\Omega)}\sum_{v\epsilon V(\Psi)}[\delta_{\Omega+\Psi}(u)^2+ \delta_{\Omega+\Psi}(v)^2]^2$$

$HF(\Omega + \Psi) = HF(\Psi) +4h_1^4g_2 +8h_1^2F_1(\Psi) HF(\Omega) +HF(\Psi)+4(h_2^4g_1+h_1^4g_2)+8(h_2^2F_1(\Omega) +h_1^2F_1(\Psi)) + (M_1(\Omega)h_2+g_1h_2^3+8g_1h_2^2+ M_1(\Psi)h_1+h_1^3h_2+8h_1^2g_2)^2$.

which completes the proof.

Theorem: 2.3

Let Ω and Ψ be a graphs. Then

$$HF(\Omega \odot \Psi) = 4h_2^4g_1+ 4h_2^2F_1(\Omega)+ HF(\Omega)+h_1[4g_2+4F_1(\Psi)+HF(\Psi)] +[M_1(\Omega)h_2+h_1h_2^3+8g_1h_2^2 +M_1(\Psi)h_1+h_1h_2+8g_2h_1]^2.$$

Proof:

By definition of HF index, we have

$$HF(\Omega \odot \Psi) = \sum_{uv\epsilon E(\Omega[\Psi])}[\delta_{\Omega[\Psi]}(u)^2+ \delta_{\Omega[\Psi]}(v)^2]^2$$

$$= \sum_{uv\epsilon E(\Omega)}[2|V(\Psi)|^2+ \delta_\Omega(u)^2+\delta_\Omega(v)^2]^2+ \sum_{uv\epsilon E(\Psi)}\sum_{i=1}^{|V(\Omega)|}[\delta_\Psi(u)^2+2+ \delta_\Psi(v)^2]^2$$

$$+\sum_{u\epsilon V(\Omega)}\sum_{v\epsilon V(\Psi)}[|V(\Psi)|+\delta_\Omega(u)^2+ \delta_\Psi(v)^2+1]^2$$

From A, $\sum_{uv\epsilon E(\Omega)}[2|V(\Psi)|^2+\delta_\Omega(u)^2+\delta_\Omega(v)^2]^2 = 4h_2^4g_1+4h_2^2F_1(\Omega) + HF(\Omega)$

From B, $\sum_{uv\epsilon E(\Psi)}\sum_{i=1}^{|V(\Omega)|}[\delta_\Psi(u)^2+2+ \delta_\Psi(v)^2]^2 = h_1 [4g_2+4F_1(\Psi)+HF(\Psi)]$

From C, $\sum_{u\epsilon V(\Omega)}\sum_{v\epsilon V(\Psi)}[|V(\Psi)|+\delta_\Omega(u)^2+ \delta_\Psi(v)^2+1]^2$

$$= [M_1(\Omega)h_2+h_1h_2^3+8g_1h_2^2+ M_1(\Psi)h_1 + h_1h_2+8g_2h_1]^2$$

$$HF(\Omega \odot \Psi) = 4h_2^4g_1+ 4h_2^2F_1(\Omega)+ HF(\Omega)+ h_1[4g_2+4F_1(\Psi)+HF(\Psi)] +[M_1(\Omega)h_2+h_1h_2^3 +8g_1h_2^2 +M_1(\Psi)h_1+h_1h_2+8g_2h_1]^2.$$

which completes the proof.

Corollary: 2.4

(i) $HF(C_n) = HF(S_n) = (4n)^2$; $n\geq3$

(ii) $HF(P_n) = (4n-6)^2$; $n\geq2$

Example: 2.5

Ω	$P_2 \otimes P_3$	$P_2 [P_3]$	$P_2 \boxtimes P_3$	$P_2 + P_3$	$P_2 \times P_3$
$HF(\Omega)$	100	20,044	12,396	6,822	1128

III. Y-INDEX OF NEW GRAPH OPERATIONS RELATED TO THE JOIN OF GRAPHS

The join of graphs:

In this section we derive the Y index of vertex $\mathcal{F}$-join and edge $\mathcal{F}$-join of new graph operations related to the graphs join where $\mathcal{F}$= {S, R, Q,T}.

Definition 3.1.

Let Ω and Ψ be a two graph the vertex $\mathcal{F}$-join graph of Ω and Ψ is a graph derived from $\mathcal{F}(\Omega)$ and Ψ by joining each vertex of Ω , $I(\Omega)$ to every vertex of Ψ, the vertex and edge sets are respectively, and is defined by $\Omega \dot{V}_{\mathcal{F}} \Psi$.

Definition 3.2

Let Ω and Ψ be a two graph the edge $\mathcal{F}$-join graph of Ω and Ψ is a graph derived from $\mathcal{F}(\Omega)$ and Ψ by joining each vertex of Ω , $I(\Omega)$ to every vertex of Ψ, the vertex and edge sets are respectively, and is defined by $\Omega \underline{V}_{\mathcal{F}} \Psi$.

Vertex and edge S-join of graphs.

Let Ω and Ψ be a two vertices disjoint graph the vertex S-join graph is derived from $S(\Omega)$ and Ψ by joining each vertex of $V(\Omega)$ to every vertex of Ψ is denoted by $\Omega \dot{V}_S \Psi$.

Let Ω and Ψ be a two vertices disjoint graph the edge S-join graph is derived from $S(\Omega)$ and Ψ by joining each vertex of $I(\Omega)$ to every vertex of Ψ is denoted by $\Omega \underline{V}_S \Psi$.

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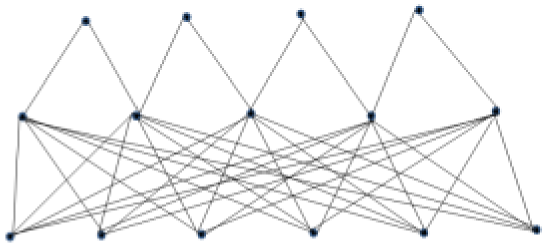
$P_5 \dot{v}_s P_6$



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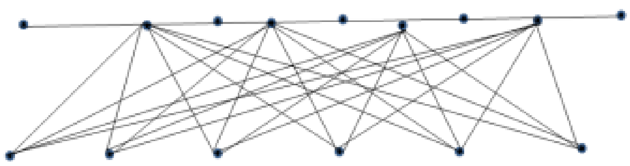


Theorem: 3.3

If Ω and Ψ be a graphs. Then

$$Y(\Omega\ \dot{v}_s\ \Psi) = Y(\Omega)+Y(\Psi)+m_2^4h_1+4F(\Omega)m_2+ 6M_1(\Omega)m_2^2+4g_1m_2^3+m_1^4h_2+4F(\Psi)m_1 +6M_1(\Psi)m_1^2+4g_2m_1^3+16g_1.$$

Proof:

By definition of Y index, we have

$$Y(\Omega\ \dot{v}_s\ \Psi) = \sum_{v\epsilon V(\Omega \dot{v}_s\Psi)}(\delta_{\Omega \dot{v}_s\Psi}(v)^4)$$

$$= \sum_{v\epsilon V(\Omega)} (\delta_{\Omega \dot{v}_s\Psi}(v)^4)+ \sum_{v\epsilon V(\Psi)}(\delta_{\Omega \dot{v}_s\Psi}(v)^4)+ \sum_{v\epsilon I(\Omega)}(\delta_{\Omega \dot{v}_s\Psi}(v)^4)$$

$$= \sum_{v\epsilon V(\Omega)} (\delta_\Omega(v)+m_2)^4+ \sum_{v\epsilon V(\Psi)}(\delta_\Psi(v)+m_1)^4+ \sum_{v\epsilon I(\Omega)} 2^4$$

$$Y(\Omega\ \dot{v}_s\ \Psi) = Y(\Omega)+ Y(\Psi)+ m_2^4h_1+ 4F(\Omega)m_2+ 6M_1(\Omega)m_2^2+ 4g_1m_2^3+m_1^4h_2+4F(\Psi)m_1 +6M_1(\Psi)m_1^2+4g_2m_1^3+16g_1.$$

which completes the proof.

Theorem: 3.4

If Ω and Ψ be a graphs. Then

$$Y(\Omega\ \underline{v}_s\ \Psi) = Y(\Omega)+ Y(\Psi)m_1^4h_2 +4F(\Psi)m_1 +6M_1(\Psi)m_1+4g_2m_1^3+(2+m_2)^4g_1.$$

Proof:

By definition of Y index, we have

$$Y(\Omega\ \underline{v}_s\ \Psi) = \sum_{v\epsilon V(\Omega \underline{v}_s\Psi)}(\delta_{\Omega \underline{v}_s\Psi}(v)^4)$$

$$= \sum_{v\epsilon V(\Omega)} (\delta_{\Omega \underline{v}_s\Psi} (v)^4)+ \sum_{v\epsilon V(\Psi)}(\delta_{\Omega \underline{v}_s\Psi} (v)^4)+ \sum_{v\epsilon I(\Omega)}(\delta_{\Omega \underline{v}_s\Psi} (v)^4)$$

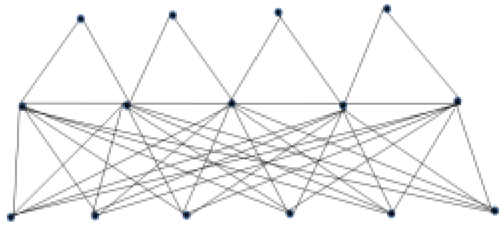
$$= \sum_{v\epsilon V(\Omega)} (\delta_\Omega(v))^4+ \sum_{v\epsilon V(\Psi)}(\delta_\Psi(v)+m_1)^4+ \sum_{v\epsilon I(\Omega)}(2+m_2)^4$$

$$Y(\Omega\ \underline{v}_s\ \Psi) = Y(\Omega)+ Y(\Psi)m_1^4h_2 +4F(\Psi)m_1 +6M_1(\Psi)m_1+4g_2m_1^3+(2+m_2)^4g_1.$$

which completes the proof.

Vertex and edge R-join of graphs.

Let Ω and Ψ be a two vertices disjoint graph the vertex R-join graph is derived from R(Ω) and Ψ by joining each vertex of V(Ω) to every vertex of Ψ is denoted by $\Omega\ \dot{v}_R\ \Psi$.
Let Ω and Ψ be a two vertices disjoint graph the edge R-join graph is derived from R(Ω) and Ψ by joining each vertex of I(Ω) to every vertex of Ψ is denoted by $\Omega\ \underline{v}_R\ \Psi$.



Theorem: 3.5

If Ω and Ψ be a graphs. Then

$$Y(\Omega\ \dot{v}_R\ \Psi) = 16Y(\Omega)+ Y(\Psi) +m_2^4h_1 +8F(\Omega)m_2 +12M_1(\Omega)m_2^2+8m_2^3g_2+h_2m_1^4 +4F(\Psi)m_1 +6M_1(\Psi)m_1^2 +4m_1^3g_1+16g_1.$$

Proof:

By definition of Y index, we have

$$Y(\Omega\ \dot{v}_R\ \Psi) = \sum_{v\epsilon V(\Omega \dot{v}_R\Psi)}(\delta_{\Omega \dot{v}_R\Psi} (v)^4)$$

$$= \sum_{v\epsilon V(\Omega)} (\delta_{\Omega \dot{v}_R\Psi} (v)^4)+ \sum_{v\epsilon V(\Psi)}(\delta_{\Omega \dot{v}_R\Psi} (v)^4)+ \sum_{v\epsilon I(\Omega)}(\delta_{\Omega \dot{v}_R\Psi} (v)^4)$$

$$= \sum_{v\epsilon V(\Omega)} (2\delta_\Omega(v)+m_2)^4+ \sum_{v\epsilon V(\Psi)}(\delta_\Psi(v)+m_1)^4 +\sum_{v\epsilon I(\Omega)} 2^4$$

$$Y(\Omega\ \dot{v}_R\ \Psi) = 16Y(\Omega)+ Y(\Psi)+ m_2^4h_1 +8F(\Omega)m_2$$

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$+12M_1(\Omega)m_2^2+8m_2^3g_2+h_2m_1^4+4F(\Psi)m_1$

$+\ 6M_1(\Psi)m_1^2+4m_1^3g_1+16g_1.$

which completes the proof.

Theorem: 3.6

If Ω and Ψ be a graphs. Then

$$Y(\Omega\ \underline{v}_R\,\Psi) = 16Y(\Omega)+Y(\Psi)+(2+h_2)^4g_1+h_2g_1^4 +4F(\Psi)g_1+6M_1(\Omega)g_1^2+8g_1^3h_2.$$

Proof:

By definition of Y index, we have

$$Y(\Omega\ \underline{v}_R\,\Psi) = \sum_{v\epsilon V(\Omega\ \underline{v}_R\,\Psi)}(\delta_{\Omega\ \underline{v}_R\,\Psi}(v)^4)$$

$$= \sum_{v\epsilon V(\Omega)}(\delta_{\Omega\ \underline{v}_R\,\Psi}(v)^4)+\sum_{v\epsilon V(\Psi)}(\delta_{\Omega\ \underline{v}_R\,\Psi}(v)^4)+$$

$$\sum_{v\epsilon I(\Omega)}(\delta_{\Omega\ \underline{v}_R\,\Psi}(v)^4) = \sum_{v\epsilon V(\Omega)}(2\delta_\Omega(v))^4+$$

$$\sum_{v\epsilon V(\Psi)}(\delta_\Psi(v)+g_1)^4+\ \sum_{v\epsilon I(\Omega)}(2+h_2)^4$$

$$Y(\Omega\ \underline{v}_R\,\Psi) = 16Y(\Omega)+Y(\Psi)+(2+h_2)^4g_1+h_2g_1^4+$$

$$4F(\Psi)g_1+6M_1(\Omega)g_1^2+8g_1^3h_2.$$

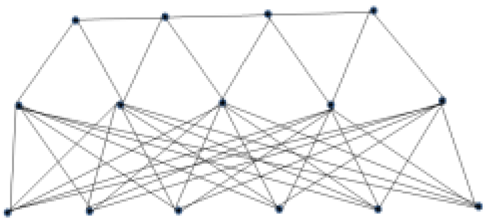
which completes the proof.

Vertex and edge Q-join of graphs.

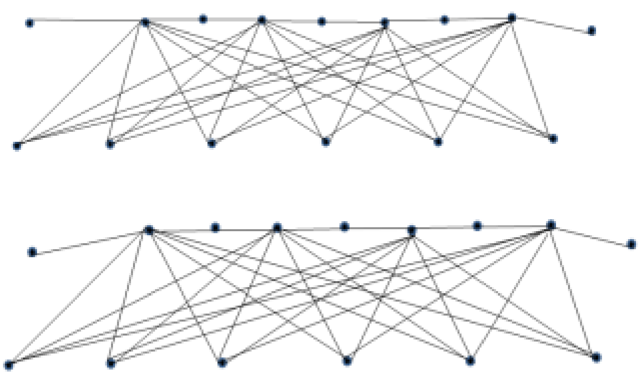
Let Ω and Ψ be a two vertices disjoint graph the vertex Q-join graph is derived from Q(Ω) and Ψ by joining each vertex of V(Ω) to every vertex of Ψ is denoted by $\Omega\ \dot{v}_Q\,\Psi$.

Let Ω and Ψ be a two vertices disjoint graph the edge Q-join graph is derived from Q(Ω) and Ψ by joining each vertex of I(Ω) to every vertex of Ψ is denoted by $\Omega\ \underline{v}_Q\,\Psi$.

$P_5\dot{v}_QP_6$



$P_5\ \underline{v}_Q\ P_6$



Theorem: 3.7

If Ω and Ψ be a graphs. Then

$$Y(\Omega\ \dot{v}_Q\,\Psi) = Y(\Omega)+Y(\Psi)+h_2^4h_1+4F(\Omega)h_2+6M_1(\Omega)h_2^2+8g_1h_2^3+h_1^4h_2+4F(\Psi)h_1+6M_1(\Psi)h_1^2+8g_2h_1^3+M_1^5(\Omega)+4Z_{3,1}(\Omega)+6R_2(\Omega).$$

Proof:

By definition of Y index, we have

$$Y(\Omega\ \dot{v}_Q\,\Psi) = \sum_{v\epsilon V(\Omega\ \dot{v}_Q\,\Psi)}(\delta_{\Omega\ \dot{v}_Q\,\Psi}(v)^4)$$

$$= \sum_{v\epsilon V(\Omega)}(\delta_{\Omega\ \dot{v}_Q\,\Psi}(v)^4)+\sum_{v\epsilon V(\Psi)}(\delta_{\Omega\ \dot{v}_Q\,\Psi}(v)^4)+$$

$$\sum_{v\epsilon I(\Omega)}(\delta_{\Omega\ \dot{v}_Q\,\Psi}(v)^4)$$

$$= \sum_{v\epsilon V(\Omega)}(\delta_\Omega(v)+h_2)^4+\sum_{v\epsilon V(\Psi)}(\delta_\Psi(v)+h_1)^4+$$

$$\sum_{uv\epsilon E(\Omega)}(\delta_\Omega(u)+\delta_\Omega(v))^4$$

$$Y(\Omega\ \dot{v}_Q\,\Psi) = Y(\Omega)+Y(\Psi)+h_2^4h_1+4F(\Omega)h_2+$$

$$6M_1(\Omega)h_2^2+8g_1h_2^3+h_1^4h_2+4F(\Psi)h_1+6M_1(\Psi)h_1^2+$$

$$8g_2h_1^3+M_1^5(\Omega)+4Z_{3,1}(\Omega)+6R_2(\Omega).$$

which completes the proof.

Theorem: 3.8

If Ω and Ψ be a graphs. Then

$$Y(\Omega\ \underline{v}_Q\,\Psi) = Y(\Omega)+Y(\Psi)+4F(\Psi)g_1+6M_1(\Psi)g_1^2+8g_1^3g_2+g_1^4h_2+M_1^5(\Omega)+h_2^4g_1+4Z_{3,1}(\Omega)+4h_2Y(\Omega)+4M_1(\Omega)h_2^3+6R_2(\Omega)+12h_2Z_{2,1}(\Omega)+6h_2^2F(\Omega)+12M_2(\Omega)h_2^2.$$

Proof:

By definition of Y index, we have

$$Y(\Omega\ \underline{v}_Q\,\Psi) = \sum_{v\epsilon V(\Omega\ \underline{v}_Q\,\Psi)}(\delta_{\Omega\ \underline{v}_Q\,\Psi}(v)^4)$$

$$= \sum_{v\epsilon V(\Omega)}(\delta_{\Omega\ \underline{v}_Q\,\Psi}(v)^4)+\sum_{v\epsilon V(\Psi)}(\delta_{\Omega\ \underline{v}_Q\,\Psi}(v)^4)+$$

$$\sum_{v\epsilon I(\Omega)}(\delta_{\Omega\ \underline{v}_Q\,\Psi}(v)^4)$$

$$= \sum_{v\epsilon V(\Omega)}(\delta_\Omega(v))^4+\sum_{v\epsilon V(\Psi)}(\delta_\Psi(v)+g_1)^4+$$

$$\sum_{uv\epsilon E(\Omega)}(\delta_\Omega(u)+\delta_\Omega(v)+h_2)^4$$

$$Y(\Omega\ \underline{v}_Q\,\Psi) = Y(\Omega)+Y(\Psi)+4F(\Psi)g_1+6M_1(\Psi)g_1^2+$$

$$8g_1^3g_2+g_1^4h_2+M_1^5(\Omega)+h_2^4g_1+4Z_{3,1}(\Omega)+4h_2Y(\Omega)+$$

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$4M_1(\Omega)h_2^3+6R_2(\Omega)+12h_2Z_{2,1}(\Omega)+6h_2^2F(\Omega)$ +

$12M_2(\Omega)h_2^2$.

which complete the proof.

Vertex and edge T-join of graphs.

Let Ω and Ψ be a two vertices disjoint graph the vertex T-join graph is derived from T(Ω) and Ψ by joining each vertex of V(Ω) to every vertex of Ψ is denoted by $\Omega\ \dot{\vee}_T\ \Psi$.

Let Ω and Ψ be a two vertices disjoint graph the edge R-join graph is derived from T(Ω) and Ψ by joining each vertex of I(Ω) to every vertex of Ψ is denoted by $\Omega\ \underline{\vee}_T\ \Psi$.

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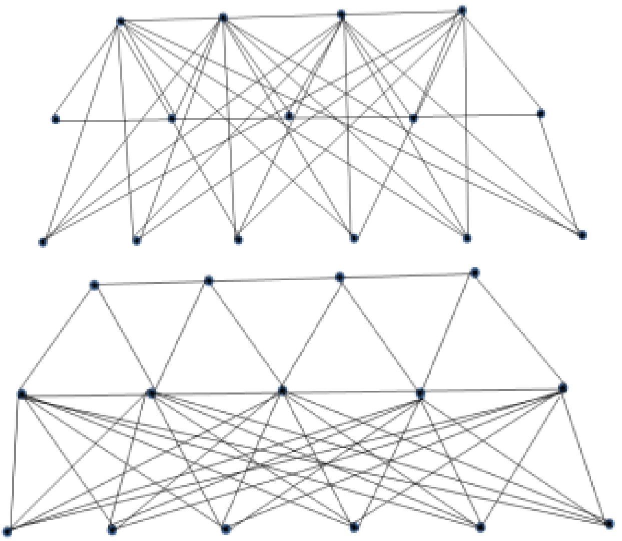

Theorem: 3.9

If Ω and Ψ be a graphs. Then

$Y(\Omega\ \dot{\vee}_T\ \Psi) = 16Y(\Omega)+Y(\Psi)+h_2^4h_1+32F(\Omega)h_2+$

$24M_1(\Omega)h_2^2+16g_1h_2^3+h_1^4h_2+4F(\Psi)h_1$

$+6M_1(\Psi)h_1^2+ 8g_2h_1^3+M_1^5(\Omega) +4Z_{3,1}(\Omega)+6R_2(\Omega)$.

Proof:

By definition of Y index, we have

$Y(\Omega\ \dot{\vee}_T\ \Psi) = \sum_{v\in V(\Omega \dot{\vee}_T \Psi)}(\delta_{\Omega \dot{\vee}_T \Psi}(v)^4)$

$= \sum_{v\in V(\Omega)} (\delta_{\Omega \dot{\vee}_T \Psi}(v)^4)+ \sum_{v\in V(\Psi)}(\delta_{\Omega \dot{\vee}_T \Psi}(v)^4)+$

$\sum_{v\in I(\Omega)}(\delta_{\Omega \dot{\vee}_T \Psi}(v)^4)$

$= \sum_{v\in V(\Omega)} (2\delta_\Omega(v)+h_2)^4+\sum_{v\in V(\Psi)}(\delta_\Psi(v)+h_1)^4+$

$\sum_{uv\in E(\Omega)}(\delta_\Omega(u)+\delta_\Omega(v))^4$

$Y(\Omega\ \dot{\vee}_T\ \Psi) = 16Y(\Omega)+Y(\Psi)+h_2^4h_1+32F(\Omega)h_2+$ $24M_1(\Omega)h_2^2+ 16g_1h_2^3+h_1^4h_2+4F(\Psi)h_1+6M_1(\Psi)h_1^2+$ $8g_2h_1^3+M_1^5(\Omega)+4Z_{3,1}(\Omega)+6R_2(\Omega)$.

which completes the proof.

Theorem: 3.10

If Ω and Ψ be a graphs. Then

$Y(\Omega\ \underline{\vee}_T\ \Psi) = 16Y(\Omega)+Y(\Psi)+4F(\Psi)g_1+ 6M_1(\Psi)g_1^2$ $+ 8g_1^3g_2+g_1^4h_2+M_1^5(\Omega)+h_2^4g_1 +4Z_{3,1}(\Omega)+ 4h_2Y(\Omega)+$ $4M_1(\Omega)h_2^3+ 6R_2(\Omega)+ 12h_2Z_{2,1}(\Omega)+6h_2^2F(\Omega)$ $+12M_2(\Omega)h_2^2$.

Proof:

By definition of Y index, we have

$Y(\Omega\ \underline{\vee}_T\ \Psi) = \sum_{v\in V(\Omega \underline{\vee}_T \Psi)}(\delta_{\Omega \underline{\vee}_T \Psi}(v)^4)$

$= \sum_{v\in V(\Omega)} (\delta_{\Omega \underline{\vee}_T \Psi}(v)^4)+ \sum_{v\in V(\Psi)}(\delta_{\Omega \underline{\vee}_T \Psi}(v)^4)+$

$\sum_{v\in I(\Omega)}(\delta_{\Omega \underline{\vee}_T \Psi}(v)^4)$

$= \sum_{v\in V(\Omega)} (2\delta_\Omega(v))^4+ \sum_{v\in V(\Psi)}(\delta_\Psi(v)+g_1)^4+$

$\sum_{uv\in E(\Omega)}(\delta_\Omega(u)+\delta_\Omega(v)+h_2)^4$

$Y(\Omega\ \underline{\vee}_T\ \Psi) = 16Y(\Omega)+Y(\Psi)+4F(\Psi)g_1+6M_1(\Psi)g_1^2+$

$8g_1^3g_2+g_1^4h_2+M_1^5(\Omega)+h_2^4g_1 +4Z_{3,1}(\Omega) +4h_2Y(\Omega)+$

$4M_1(\Omega)h_2^3+6R_2(\Omega)+12h_2Z_{2,1}(\Omega)+6h_2^2F(\Omega)$

$+12M_2(\Omega)h_2^2$.

which completes the proof.

IV. CONCLUSIONS

HF indices are a pair of recently introduced graph invarients that generalize much used F index. In this paper, we obtain some explicit expression of the HF index of some different graph operations such as join, Cartesian product, corona product of graphs. We compute some useful formula of the y index of graphs based on the vertex and edge $\mathcal{F}$-join of graphs.

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