International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-11 Issue-2, December 2021

Relaxed Skolam Mean Labeling of 6 – Star Graphs with Partition 3, 3

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***Abstract**: Existence Relaxed skolam mean labeling for a 6 – star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2} \cup K_{1,\beta_3}$ with partition 3,3 with a certain condition is the core topic of the following article. Trial and error method is used to find the existence of the relaxed skolam mean labeling of 6 - star graph with partition 3, 3 holding a specific condition.*

***Keywords**: Star Graphs, Union of Star Graph, Labeling, Skolem Mean Labeling, Relaxed Skolam Mean Graph.*

I. INTRODUCTION

The concept of labeling in Graphs plays a vital and undeniable role in the field of planning and networking. Some of the most import labeling functions which are discovered far more earlier and famous were graceful labeling, prime labeling, cordial labeling, mean labeling etc.. In this article we discuss a type of labeling namely Relaxed Skolam Mean Labeling which is extracted from Skolam Mean Labeling of Graphs introduced by V. Balaji et.al.[5] in the year 2010 which in turn is derived from the mean labeling of Graphs .

II. PRELIMINARIES

Definition 2.1 [5]: A graph $G=(V, E)$ with p vertices and q edges is said to be a relaxed skolam mean graph if there exists a function $f : V \rightarrow \{1, 2, 3, ..., p+1 = |V| + 1\}$ such that the induced edge map $f^* : E \rightarrow \{2, 3, ..., p = |V| + 1\}$ given by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if } (f(u) + f(v)) \text{ is even} \\ \dfrac{f(u) + f(v) + 1}{2} & \text{if } (f(u) + f(v) + 1) \text{ is even} \end{cases}$$

The resulting distinct edge labels are from the set $\{2, 3, ..., p+1 = |V| + 1\}$

Note: There are p vertices and available vertex labels are p + 1 and hence one number from the set $\{1,2,3,...,p+1=|V|+1\}$ is not used and we call that number as the relaxed label. When the relaxed label is p + 1, the relaxed mean labeling becomes a skolam mean labeling.

Result: The three star graph $K_{1,a} \cup K_{1,b} \cup K_{1,c}$ satisfies relaxed skolam mean labeling if $a + b \le c \le a + b + c$.

III. MAIN RESULT

Theorem: The 6 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2} \cup K_{1,\beta_3}$ where $\alpha_1 \le \alpha_2 \le \alpha_3$ and $\beta_1 \le \beta_2 \le \beta_3$ is a relaxed skolam mean graph if $\beta_1 + \beta_2 + \beta_3 - \alpha_1 - \alpha_2 - \alpha_3 = 7$.

Proof: Let $\sigma_1 = \alpha_1; \sigma_2 = \alpha_1 + \alpha_2; \sigma_3 = \alpha_1 + \alpha_2 + \alpha_3$ and $\delta_1 = \beta_1; \delta_2 = \beta_1 + \beta_2; \delta_3 = \beta_1 + \beta_2 + \beta_2 + \beta_3$.

Consider the 6 - star graph

$G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2} \cup K_{1,\beta_3}$.

The condition $\beta_1 + \beta_2 + \beta_3 - \alpha_1 - \alpha_2 - \alpha_3 = 7$ gives rise to the case $\delta_3 = \sigma_3 + 7$ In this case we will establish that the graph G is relaxed skolam mean.

Let the set of vertices of G be $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 \cup V_6$ where

Manuscript received on November 29, 2021.
Revised Manuscript received on December 01, 2021.
Manuscript published on December 30, 2021.
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$V_k = \{v_{k,i} : 0 \le i \le \alpha_k\}; 1 \le k \le 3$ and

$V_4 = \{v_{4,i} : 0 \le i \le \beta_1\}$; $V_5 = \{v_{5,i} : 0 \le i \le \beta_2\}$; $V_6 = \{v_{6,i} : 0 \le i \le \beta_3\}$. Let the edge set of G be $E = \bigcup_{k=1}^{3} \{v_{k,0}v_{k,i} : 1 \le i \le \alpha_k\} \cup \bigcup_{k=4}^{6} \{v_{k,0}v_{k,i} : 1 \le i \le \beta_{k-3}\}$.

Case: Let $\delta_3 = \sigma_3 + 7$

G has $\sigma_3 + \delta_2 + 6 = 2\sigma_3 + 13$ vertices and $\sigma_3 + \delta_2 = 2\sigma_3 + 7$ edges.

We define the relaxed skolam vertex function

$f : V \to \{1, 2, ..., p+1 = \sigma_3 + \delta_2 + 6 + 1 = 2\sigma_3 + 14\}$ as follows:

$f(v_{1,0}) = 1;\quad f(v_{2,0}) = 3;\ f(v_{3,0}) = 5;$

$f(v_{4,0}) = \sigma_3 + \delta_3 + 5 = 2\sigma_3 + 9;$

$f(v_{5,0}) = \sigma_3 + \delta_3 + 6 = 2\sigma_3 + 11;$

$f(v_{6,0}) = \sigma_3 + \delta_3 + 6 = 2\sigma_3 + 13$

$f(v_{1,\kappa}) = 2\kappa + 5 \qquad 1 \le \kappa \le \alpha_1$

$f(v_{2,\kappa}) = 2\sigma_1 + 2\kappa + 5 \qquad 1 \le \kappa \le \alpha_2$

$f(v_{3,\kappa}) = 2\sigma_2 + 2\kappa + 5 \qquad 1 \le \kappa \le \alpha_3$

$f(v_{4,\kappa}) = 2\kappa \qquad 1 \le \kappa \le \beta_1$

$f(v_{5,\kappa}) = 2\delta_1 + 2\kappa \qquad 1 \le \kappa \le \beta_2$

$f(v_{6,\kappa}) = 2\delta_2 + 2\kappa \qquad 1 \le \kappa \le \beta_3$

Here the relaxed label is $2\sigma_3 + 6$

The edge labels are given as follows:

The edge labels of $v_{1,0}v_{1,\kappa}$ is $\kappa + 3$ for $1 \le \kappa \le \alpha_1$ ($2, 3, ..., \alpha_1 + 1 = \sigma_1 + 1$), $v_{2,0}v_{2,j}$ is $\sigma_1 + \kappa + 4$ for $1 \le \kappa \le \alpha_2$ ($\sigma_1 + 5, \sigma_1 + 6, ..., \sigma_1 + \alpha_2 + 4 = \sigma_2 + 4$), $v_{3,0}v_{3,j}$ is $\sigma_2 + \kappa + 5$ for $1 \le \kappa \le \alpha_3$ ($\sigma_2 + 6, \sigma_2 + 7, ..., \sigma_2 + \alpha_2 + 5 = \sigma_3 + 5$), $v_{4,0}v_{4,j}$ is $\sigma_3 + \kappa + 5$ for $1 \le \kappa \le \beta_1$ ($\sigma_3 + 6, \sigma_3 + 7, ..., \sigma_3 + \beta_1 + 5 = \sigma_3 + \delta_1 + 5$), $v_{5,0}v_{5,\kappa}$ is $\sigma_3 + \delta_1 + \kappa + 6$ for $1 \le \kappa \le \beta_2$ ($\sigma_3 + \delta_1 + 7, \sigma_3 + \delta_1 + 8, ..., \sigma_3 + \delta_1 + (\beta_2) + 6 = \sigma_3 + \delta_2 + 6$) , $v_{6,0}v_{6,\kappa}$ is $\sigma_3 + \delta_2 + \kappa + 7$ for $1 \le \kappa \le \beta_3$ ($\sigma_3 + \delta_2 + 8, \sigma_3 + \delta_2 + 9, ..., \sigma_3 + \delta_2 + (\beta_3) + 7 = \sigma_3 + \delta_3 + 7 = 2\sigma_3 + 13$)

The edge labels are

$2, 3, ..., \sigma_1 + 1$

$\sigma_1 + 5, \sigma_1 + 6, ..., \sigma_2 + 4$

$\sigma_2 + 6, \sigma_2 + 7, ..., \sigma_3 + 5$

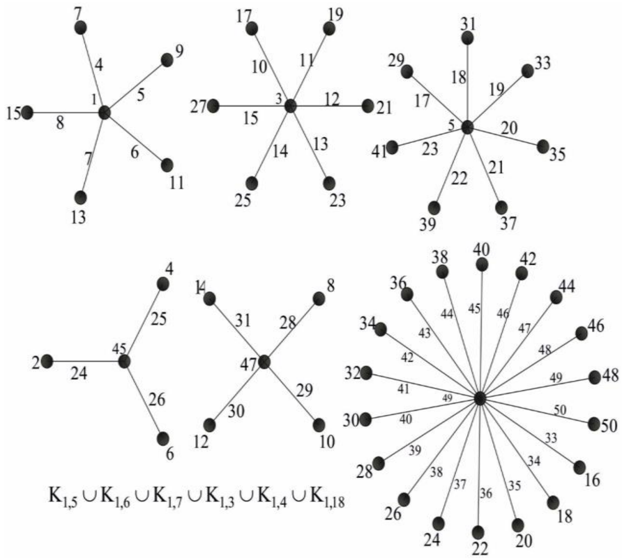
$\sigma_3 + 6, \sigma_3 + 7, ..., \sigma_3 + \beta_1 + 5 = \sigma_3 + \delta_1 + 5$

$\sigma_3 + \delta_1 + 7, \sigma_3 + \delta_1 + 8, ..., \sigma_3 + \delta_1 + (\beta_2) + 6 = \sigma_3 + \delta_2 + 6)$

$\sigma_3 + \delta_2 + 8, \sigma_3 + \delta_2 + 9, ..., \sigma_3 + \delta_2 + (\beta_3) + 7 = \sigma_3 + \delta_3 + 7 = 2\sigma_3 + 13)$

The images of the relaxed skolam edge function of the graph G are distinct. Hence G is a relaxed skolam mean graph.

Example:



$K_{1,5} \cup K_{1,6} \cup K_{1,7} \cup K_{1,3} \cup K_{1,4} \cup K_{1,18}$

IV. CONCLUSION

In this research article we concentrated mainly on the existence of relaxed skolam mean labeling of a 6 - star graph $G = K_{1,\alpha_1} \cup K_{1,\alpha_2} \cup K_{1,\alpha_3} \cup K_{1,\beta_1} \cup K_{1,\beta_2} \cup K_{1,\beta_3}$ with the condition $\beta_1 + \beta_2 + \beta_3 - \alpha_1 - \alpha_2 - \alpha_3 = 7$. Trial and error method is used to find the existence of the labeling function.

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Retrieval Number: 100.1/ijeat.B32811211221
DOI: 10.35940/ijeat.B3281.1211221
Journal Website: www.ijeat.org

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

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