International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-10 Issue-3, February 2021

Computation of Odd Geo-Domination Number of a Graph

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***Abstract:** A geodominating set $S \subseteq V$ of a graph G is said to be an odd geo-dominating set if for every vertex $v \in V - S$, $v \in V - S$, $N(v) \cap S \neq \emptyset$ and $|N(v) \cap S| \equiv 1 (mod\ 2)$ The minimum cardinality of the odd geo-dominating set is called odd geo-domination number denoted by $g_{odd}(G)$. The odd geo-dominating set with cardinality $g_{odd}(G)$ is called $g_{odd}-$ set of G. We develop an algorithm to compute an odd geo-domination number of graphs and for some families of graphs.*

***Keywords:** geodesic, geodominating set, geodomination number, odd -geodomination number, graph algorithm*

I. INTRODUCTION

There are interesting applications of geodetic number concepts [3] to the problem of designing the route for a shuttle and network. The different other areas that apply geodetic number concepts are telephone switching centers, facility location, distributed computing, image and video editing, neural networks and data mining. The geodetic number of a graph was introduced in [3, 8] and further studied in [5, 6]. In [6], geodetic sets and geodetic number were referred to as geodominating sets and geodomination number and this terminology is used throughout this paper. Paper [10] motivates us to develop the new definition odd geo-domination number [2]. In this paper we have developed an algorithm to find odd geo-domination number of a graph. Basic tools to develop these algorithms were developed in [1].

II. PRELIMINARIES

Definition 2.1: [2] A geodominating set $S \subseteq V$ of a graph G is said to be an odd geo-dominating set if for every vertex $v \in V - S$, $N(v) \cap S \neq \emptyset$ and $|N(v) \cap S| \equiv 1 (mod\ 2)$. The minimum cardinality of the odd geo-dominating set is called odd geo-domination number denoted by $g_{odd}(G)$. The odd geo-dominating set with cardinality $g_{odd}(G)$ is called $g_{odd}-$ set of G.

Manuscript received on December 26, 2020.
Revised Manuscript received on January 06, 2021.
Manuscript published on February 28, 2021.
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Definition 2.2: **[1]** Characterize each closed interval as a n-tuple. Each place of n-tuple can be represented by a binary 1 or 0. Call this n-tuple as a *link vector*. Denote $LV(I) = I'$. Put 1 if the vertex belongs to the closed interval otherwise 0. If all the co-ordinate of the link vector are equal to 1 then it is called as *full*. Denote I [(1)].

Theorem 2.3: **[2]** For any connected graph G, $2 \leq g(G) \leq g_{odd}(G) \leq n$.

Algorithm 2.4

Algorithm: Geodominating set confirmation

Procedure geodetic [S]

Input: A graph $G = (V, E)$, $V = \{v_1, v_2, \dots, v_n\}$, a subset $S = \{v_1, v_2, \dots, v_k\}$

Output: S is geodominating set or not.

begin

Step 1: Find all the 2-subsets S_2 of S

{There are $\binom{k}{2}$ number of subsets S_2 of S}

Step 2: Take $L \leftarrow (0)$

Step 3: for $i = 1$ to $\binom{k}{2}$

begin

Closed- interval $I_i[S_2]$

Link vector $I'_i[S_2]$

$L = L \vee I'_i[S_2]$

end

Step 4: If L is full, then Print: "The given set S is geodetic", Stop.

Otherwise, Print: "The given set S is not geodetic",

End

In this algorithm, step 2 will work in $\frac{k(k-1)}{2}$ times.

Next part of step 2 is the algorithm Closed-interval $I_i[S_2]$ and Link vector $I'_i[S_2]$ and hence this part will work with $2m+n$ verifications. Total cost of time is $O(k^2(m+n))$, where k is the cardinality of the given vertex subset and m is the number of edges in G. But in this step the given vertex acts as a root and all other vertices are approached through a spanning tree. Therefore there are $n+(n-1)$ verifications needed, since $m = n-1$ for a tree. That is, $O(n^2(2n-1))$, that is $O(n^3)$. Thus this algorithm requires $O(n^3)$ cost of time, finally we develop the main algorithm to find the minimum geodetic set of a graph G.

III. MAIN RESULTS

In this section, we define the algorithms to compute odd geo-domination in a graph G

Algorithm 3.1:

Algorithm: Set $N(v_i)$

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication

To find $N(v_i)$ for a vertex v_i:
Input: A graph $G = (V, E)$, adjacency matrix $\{a[i, j]\}$ and the vertex v_i
Output: $N(v_i)$
Step 1: Take $N(v_i) = \emptyset$
Step 2: for $j = 1$ to n do begin
if $a[i, j] = 1$, then $N(v_i) = N(v_i) \cup \{v_j\}$
end.
Here the algorithm takes n verifications. That is, it works $O(n)$ cost of times.

Algorithm 3.2
Algorithm: Compute $|N(v_i) \cap S|$
To find $|N(v_i) \cap S|$; $v_i \in V - S$
Input: A graph $G = (V, E)$, $V = \{v_1, v_2, \dots, v_n\}$, $v_i \in V - S$: $S = \{v_1, v_2, \dots, v_k\}$ and degree sequence $\{d(v_i)\}$.
Output: $|N(v_i) \cap S|$
Step 1: Set $N(v_i)$
Step 2: Take count = 0
Step 3: for $j = 1$ to $d(v_i)$
if $N(v_i) \in S$, then count = count + 1
Step 4: Print $|N(v_i) \cap S| =$ count
By step 1, it takes n verifications. By Step 3, it works in deg (v_i) number of times to check whether the neighborhood of v_i belongs to S or not. That is, totally it works in deg $(v_i) = k$ (say) times. Thus it requires $O(n + k)$ cost of time.

Algorithm 3.3
Procedure odd geo-dominating set [S]
Input: A graph $G = (V, E)$, $V = \{v_1, v_2, \dots, v_n\}$, $S = \{v_1, v_2, \dots, v_k\}$ and $V - S = \{v_{k+1}, v_{k+2}, \dots, v_n\}$
Output: S is odd geo-dominating or not.
Step 1: procedure geodetic [S];
Step 2: if S is geodetic, then
for $i = k + 1$ to n
procedure [$|N(v_i) \cap S|$]
Step 3: if $|N(v_i) \cap S| \bmod 2 \equiv 0$, then S is not an odd geo-dominating set.
Stop.
Otherwise S is an odd geo-dominating set.
In step 1, we call Producer geodetic [S], it requires $O(n^3)$ cost of time. Next part of the algorithm is for loop which works for $n - k$ times. In step 2, the Producer $|N(v_i) \cap S|$, it it requires $O(n + k)$ cost of time. Thus it requires $O(n^5)$ cost of time.
Algorithm 3.4
Algorithm to find minimum odd geo-dominating set:
Input: A graph $G = (V, E)$ with its vertex set $V = \{v_1, v_2, \dots, v_n\}$.
Output: g_{odd}- set of G.
Step 1: Take $k \leftarrow 2$
Step 2: Take all the subsets S_j $\left(1 \le j \le \binom{n}{k}\right)$ of V with k vertices.
Step 3: for $j = 1$ to $\binom{n}{k}$
begin
procedure odd geo-dominating set [S_j]
if yes then stop and print S_j is a g_{odd}- set of G.
end
Step 4: Otherwise take $k = k + 1$ and return to step 2.
End.
In this algorithm, by the for loop it takes all the 2^n subsets and each time it calls the procedure Odd Geo-dominating [S], it's complexity is $O(n^5)$. Hence this algorithm is NP-complete.

IV. CONCLUSION

Algorithms to compute odd geo-domination number of a graph is discussed in this paper. In further research we will try to reduce the complexity of minimum odd geo-domination number.

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